AN INVESTIGATION OF THE APPLICATION OF PROGRAMMING TECHNIQUES TO FARM MANAGEMENT PROBLEMS

With Special Reference to Beef Cattle Feeding in the Lothians of Scotland and in Alberta

by

FENTON VINCENT MacHARDY

AUTHOR

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CONTENTS

4. CONCLUSIONS

5. REFERENCES

6. APPENDICES

1. INTRODUCTION

The Lothians of Scotland are generally farmed intensively, with arable farming predominating in the more easterly regions. Potatoes form an integral part of the cropping pattern with sugar beets and turnips somewhat less widely grown. Wheat and barley are the principal cereal crops. Grass is included in the rotation on practically every farm. The returns from cropping are usually complemented by returns from the production of beef cattle. Farmers of the area usually have one or more beef cattle enterprises, frequently with emphasis on fattening cattle for slaughter.

The farming pattern, with emphasis on potato production, requires a substantial labour force, measured on an "acres per man" basis, as compared to less intensively farmed areas such as the unirrigated portions of Alberta. The availability of this labour force during non-critical seasons might be expected to result in a level of mechanization differing from that developed for less intensively farmed areas. For field work, a larger number of smaller capacity machines might be expected, and for cattle feeding a lower level of mechanization might be expected. Despite these expectations there is a keen interest in highly mechanized processes. This is particularly evident in processes associated with cattle production.

This intensive, well integrated pattern of agriculture offers a challenging application for linear programming as a management aid. In addition, the interaction between cropping practise, manpower availability, and level of mechanization, particularly with respect to beef cattle feeding, offers an excellent medium for the extension of linear programming into the fixed-cost sector of farm business.

Tabulated, the objectives of the research leading to the preparation of this thesis were to:

1) Investigate the application of linear programming as an aid to farm planning in the Lothians.

2) Complementary to this objective, to assess the availability and suitability of Scottish agriculture research data for the preparation of linear programmes.

3) The development of extensions to linear programming to permit the analysis of any mechanized activity in full economic association with other farm activities.

4) An investigation of the use of linear programming, with extensions, as a method for determining the optimum levels of mechanization for beef cattle feeding.

2. REVIEW OF LITERATURE

2.1 GENERAL DELINEATION

Each of the fields of mechanization and business planning constitutes a major area of interest to both industry and agriculture. As such, each has traditionally been an area of great activity in the production of technical publications.

In industry, the fundamental relationships between man and his production machines were studied and quantified by Fred Taylor[38] and Frank and Lillian Gilbreth[19]. The fields pioneered by Taylor and the Gilbreths, time study and motion study, have grown into the business activity generally called production, or industrial engineering. Equipment selection has become a recognized division of production. The Production Handbook[8] devotes a chapter each to Plan Layout and Location, Tools Jigs and Fixtures, Materials Handling, and Machinery and Equipment Economics. George[18] develops several criteria for equipment replacement. Terborg[39,40] has devoted a great amount of time and energy to the problem of equipment selection and replacement. His MAPI Replacement Method constitutes a major contribution to this field.

Industries' interest in equipment selection and replacement has had its counter part in agriculture. Barger[2] et al devote a chapter to "Some Economic Aspects of Farm Buildings". Culpin[12] provides data to aid in equipment selection. Section 6 of "The Farm as a Business"[24] is entitled "Aids to Management, Labour and Machinery".

These sources treat equipment selection and replacement as basically a budgeting problem. Richey[35] introduces probability of mechanical failure as a factor in sequenced operations. Link and Bockhop[26] develop a mathematical model for machinery scheduling where the requirements of the farm and the constraints of environmental conditions are imposed upon the system. Wiser[42] and MacHardy[31] develop weather simulation by the Monte Carlo technique. The use of Lagrange multipliers in minimizing convex functions representing least-cost machinery combinations is developed by MacHardy[30]. Peart[34] uses integer programming for the selection of components for a materials handling system.

In the field of farm management, specialists of the Department of Economics, Edinburgh School of Agriculture, use account analysis and the gross marging technique[33] for farm planning. Linear program is being widely tried as an aid in farm management extension. Program planning[10] is claimed to offer a manual alternative. Quadratic programming[27] extends programming into the area of non-linear relationships. It has been shown that economic criteria can be employed in making use of agricultural production functions[23] to make decisions that benefit farmers. Game theory[21] has been applied to some aspects of farm planning. Dynamic programming[41], based on the notion of recursion, offers a method of planning under conditions of change.

The profusion of methods and the variety of viewpoints presented by these writers clearly illustrates that there is no single all-encompassing method for the solution of management problems. In commencing this thesis it was decided that a method should be used that could utilize much of the existing data that has been collected by research workers in the various fields of agricultural science. It was decided also that, as it was hoped that the results of the thesis would have practical extension applications, to deal with characteristics of problems in terms most familiar to farmers. Coupling these objectives with the requirement that, insofar as possible, the entire farming operation should be presented as a single formulation, led to the conclusion that linear programming should be used as the method of analysis.

2.2 LINEAR PROGRAMMING

So much has been written about the theory and application of linear programming since its development following World War II that any list of references must necessarily be incomplete--indeed contributions to linear programming have been made by people with such a variety of interests that even the breadth of coverage must be restricted.

The literature referred to here was selected as having contributed to the coverage in depth of these aspects of the subject.

a) The history of linear programming and its development as a farm management method.

b) The relationship of the gross margin technique to linear programming.

c) Post optimal sensitivity analysis.

d) Extensions that increase the range of application of linear programming.

The relationship to each of these aspects of this thesis is probably apparent. The background of work carried out in directly related fields is included in (a). (b) develops the relationship of an aid to farm management, widely used in Scotland, to linear programming. (c) is concerned with the amount of information that may be obtained from the solution to a problem. This aspect, although not developed in this thesis, is complementary to all linear programming activity. Extensions are needed when problems cannot be formulated in terms of the linear programming mathematical model. This thesis develops extensions to meet some special problems of mechanization. However, the principal extensions (such as quadratic programming) that have been developed are not used. As some extensions (such as decomposition) could be considered complementary to all linear programming, mention of them is included in this literature review.

2.2.1 HISTORY AND DEVELOPMENT OF LINEAR PROGRAMMING

Hadley[20] credits George Dantzig with formulating the general linear programming problem and developing the simplex method for its solution. He names Charnes[9] as devising the perturbation technique for resolving the degeneracy problem. Dantzig[13] follows through the origins and influences by providing a linear programming timetable. The influences of the military, the economic-industrial and the mathematical, both theoretical and computational, are shown. The 19th century mathematicians Fourier (1823) and Gauss (1826) are shown as having probably been aware of its potential. Dantzig's timetable is useful also in showing that the method is still being developed and extended. The chronologic order of the development of integer programming, uncertainty, network theory, quadratic programming and the decomposition principle is shown.

The first record of an application of the simplex method to the solution of a practical problem is of J. Laderman of the National Bureau of Standards in 1947 solving a problem in nutrition formulated two years earlier by G.J. Stigler[36]. Since that time hundreds of agricultural problems have been formulated and solved. Candler and Musgrove[7] prepared a partial bibliography of technical literature in 1961 listing 52 contributions. Eisgruber and Reisch[16] conducted a survey in 1961 to determine the extent to which linear programming had been used by 47 land grant colleges in the United States.

The results of their survey are shown in Fig. 2.1.

How linear programming is utilized	Number of Departments
Used in research	41
Used in extension	14
Planned to be used in extension	7
Taught--	
(a) in a course specifically designed for it	12
(b) as part of a course the prime objective of which is other than teaching linear programming	31

Fig. (2.1)

The subject matter areas of application are shown in Fig. 2.2.

Subject matter area	Number of Departments
Farm management	40
Agricultural marketing	24
Agricultural production economics	6
Agricultural policy	4
Consumer economics	2
Other	8

Fig. (2.2)

Eisgruber and Reisch concluded that, while the technique of linear programming was widely used and extensively taught, the potential of the technique in agricultural economic research and application had not been anything like fully explored.

Of the many contributors to the advancement of linear programming as a farm management planning method, several have published papers that were of particular interest in the preparation of this thesis. Heady and Candler[22] develop many of the problems of data preparation for farm programming in addition to providing an economic interpretation of the simplex algorithm. MacFarquar, Barnard and associates in several publications[4,5,28] develop a general program format that maintains continuously variable relationships over a wide area of application. In addition, MacFarquar[29] has developed useful analytic methods for handling step functions, particulary those associated with crew size problems. Mills[32] developed an analysis of marginal value products for step function approximations. Armstrong and Ferris[1] provide an example of the value of linear programming in the selection of tractor and equipment combinations in California. Peart[34] has dealt with developments in integer programming as a method of handling some aspects of building and machinery selection in an integrated model.

2.2.2 THE GROSS MARGIN TECHNIQUE OF FARM PLANNING

Many of the concepts that underlie the interpretation and presentation of data for linear programming are also fundamental to the Gross Margin Technique. As in fact the gross margin technique constitutes one phase of linear programming, it is logical that a discussion of the gross margin technique should precede any discussion of linear programming.

Peart and Rowbottom[33] give credit to V. Liversage of Northern Ireland for early use of this technique in farm planning, and to the B.B.C. and Farm Economics Branch, Cambridge University, for stimulating interest in the technique through a series of television programs in 1961, and the subsequent publication of the bulletin "Planning for Profits". The logic of the gross margin approach, as developed by Peart and Rowbottom, lies in the fact that there are two types of cost items associated with the farm business, a) those costs clearly identified with individual enterprises, and b) those costs that are part of the farming system as a whole, and not readily identifiable with any particular enterprise. The first are known as <u>variable-costs</u> and the second are known as <u>fixed-costs</u>. Although there are other designations for these costs (e.g. specific costs and overhead) the terms variable-cost and fixed-cost are used throughout this thesis in the context developed for the gross margin approach.

"Gross margin" is defined as the difference between output, on one hand, and variable-costs on the other hand. Gross margin is then a measure of the contribution that an enterprise makes towards meeting the fixed-costs of the whole farm, or towards the net profit of the farm.

Expressed mathematically:

$$G_i = (R_g)_i - (C_v)_i \qquad \text{-----------}(2.1)$$

and

$$P = \sum_{i=1}^{n} G_i - \sum_{j=1}^{m} (C_f)_j \qquad \text{-----------}(2.2)$$

Where:

G = gross margin

R_g = gross returns

C_v = variable-cost

C_f = fixed-cost

P = net profit

Peart and Rowbottom list the use of the gross margin technique as:

a) an analytic tool to measure the contributions of individual enterprises towards fixed-costs and profits.

b) a means of measuring enterprise efficiency by comparing with standard data.

c) a forward planning tool to guide in the choice of enterprises, and the scale of each.

The third use of the gross margin technique provides an excellent definition of the value of linear programming. Whereas inductive reasoning may lead to a good combination of enterprise and a suitable scale for each, linear programming offers a precise mathematical tool for determining enterprise combinations and scale. The gross margins, calculated in precisely the same manner as for the gross margin method, provides the initial marginal value products, or cost-row coefficients for linear programming.

2.2.3 POST OPTIMALITY AND THE DUAL PROBLEM

Linear programming can be reduced to finding the values of

$x_1 \geq o$, --- $x_n \geq o$, and Min z, satisfying

$$Z = \sum_{j=1}^{n} C_j Y_j$$

and

$$R_i = \sum_{j=1}^{n} a_{ij} Y_j$$

$$i = 1 \text{ to } m$$

It can be shown[21,22,23] that a solution may be obtained by taking m of the x's and solving the resulting m x m system of equations. It can also be shown that one of the solutions so obtained satisfies $x_1 \geq o$, --- $x_n \geq o$ and Min Z.

Examination will show that the number of possible solutions for a m x n system of equations taking the x's m at a time is:

$$\frac{n!}{m!(n-m)!}$$

This rules out the practicability of obtaining a solution by enumerating all of the possibilities. Linear programming as a practical possibility came into being when Dantzig created the simplex method. The simplex method[23] provides a stepping-stone technique whereby the solution minimizing the objective function can be reached in a practical number of steps.

The information obtained directly from the optimal solution is the value of the objective function, the variables appearing in the final basis, and the activity of each.

It is often desirable that a sensitivity analysis should accompany the results of a practical linear program for the purpose of supplying additional information about the solution. Whereas little use has been made of sensitivity analysis in analysing the results of the practical programs developed for this thesis, it should be pointed out that such an analysis could be carried out. The literature on linear programming develops this subject in depth. The first source of additional information comes from the shadow prices. The final cost-row entries for the non-basic variables are often referred to as shadow prices. Shadow price may be defined in economic terms as -- "the penalty to the total system if a unit of this variable is forced into the final solution". It should be observed that when applied to disposal activities (slack variables) the shadow prices show the value of an increase of one unit of the resource associated with the disposal activity.

Hadley[20] lists six specific post-optimality questions that may be answered by some additional computation.

1. How much can the price vector be changed in some specific way before the optimal solution obtained will no longer be optimal?

2. For a given change in the price vector how is a new optimal solution obtained if the original solution is no longer optimal?

3. How much can the requirement vector (right hand side) be changed in some specific way before the optimal solution will no longer be feasible?

4. If a given change in the requirement vector makes the optimal solution no longer feasible, how is a new optimal solution obtained?

5. How can another variable be accounted for?

6. How can the insertion of an additional constraint be incorporated into the system?

These questions may all be answered by some additional computation. The amount of computation is generally kept to a minimum in answering these questions by making use of the dual problem* at some stage in the solution.

Considerable differences exist in the amount of post optimal information that may be obtained from various routines. Fig. 2.3 is an example of the readout obtained from a Shrimplex[17] program for use with a Ferranti Sirius computer. There is little post-optimality information. Fig. 2.4 is an example of the read-out obtained from a program[25] prepared for use with an IBM 1620 computer.

SPECIMEN RESULTS

Shrimplex

SAMPLE MIXTURE

1	1	5	-0.500000	-1.000000
2	3	6	0.000000	-2.000000
3	2	3	0.000000	-0.800000

FUNCTIONAL

		-0.800000			
X	1	0.200000			
X	2	0.800000			
			S	3	0.000000
X	0	-0.800000			
A	1	-6.000000			
X	3	6.000000			
A	2	0.400000			

Fig. 2.3

* Associated with every linear program is another linear program called the dual. The object of the original program (defined by Dantzig as the primal) is to minimize; the object of the dual is to maximize. The dual program is normally obtained by transposing the coefficient matrix, interchanging the role of the constant terms (right hand side) and the coefficients of the objective form (functional), changing the direction of inequality, and maximizing instead of minimizing. The existence of the dual permits a choice--any program may be solved by writing it in either its primal or its dual form. A saving in computer storage space may often be effected by invoking this choice.

Typewriter output

CASE 2 x 3

ITER NO	FUNCTIONAL	VAR OUT	VAR IN
001	5140000000	9992010000	0002001000
002	5130000000	9901000000	9902000000
003	5160000000	0002001000	0001001000
FUNCTIONAL	5160000000		

VAR/COST	ACTIVITY	LIM VAR	LOWER LIM	LIM VAR	UPPER LIM
9902000000	10.0000	0002	.3333-	9992	10.0000
0001001000	6.0000	0002	.5000	0000	9999.9000

VAR/COST	SHAD PRICE	LIM VAR	LOWER LIM	LIM VAR	UPPER LIM
0002001000	1.0000	0000	9999.9000-	0001	3.0000
9992010000	10.0000	9902	10.0000-	0000	9999.9000
9901000000	1.0000	0000	9999.9000-	0001	6.0000

Fig. 2.4

In addition to the information available directly from the print-out the IBM program includes cost changer and right-hand side changer sub-routines. This program, therefore, goes a long way towards providing answers to Hadley's questions.

2.2.4 LINEAR PROGRAMMING EXTENSIONS

The method of linear programming has frequently been criticized on three counts:

a) Capacity requirements for practical problems may exceed the capacity of available computers.

b) It is difficult to include the factor of risk or uncertainty.

c) The method requires linear relationships.

Although the ramifications of (a) and (b) have not been explored at all in this thesis, and those of (c) have been explored for particular problems only, it is felt that references to linear programming extensions designed to meet these criticisms should be listed.

2.2.4.1 Decomposition

Dantzig[13] notes that practical problems occur where groups of variables are controlled by common restraints but where very few restraints and a common objective tie the groups together. Dantzig develops a technique that decomposes linear programs exhibiting this characteristic into:

a) sub-programs corresponding to its almost independent parts, and

b) a master program that ties the sub-programs together.

The penalty imposed for this decomposition is that an iterative procedure is involved calling for possibly several solutions to the master program and the sub-programs. One of the attractions for agricultural application is that many sub-programs, such as those for least cost ration formulation and land use, are repeated without change for practically every farm that is programmed. It would appear that the routine work in preparing and punching individual farm programs could be very considerably reduced. Another apparent attraction is that general purpose analog computers, normally having an insufficient number of amplifiers to handle large programs, become potentially useful.

2.2.4.2 Uncertainty

Dantzig[13] introduces the topic of uncertainty with the following paragraph:

> "In the final analysis, most applied programming problems involve uncertainty in either the technology matrix or the constant terms. The techniques discussed so far, however, do not take into account the uncertain nature of the coefficients of the program. In the period 1955-60, various individuals have tried to extend linear programming methods to deal with the problem of optimizing in some sense an objective function, subject to constraints whose constants are subject to random variations (Dantzig, 1955-1; Gerguson and Dantzig, 1956-1). One of the basic difficulties is that the problem is capable of many formulations, with only fragmentary results for each of the formulations (Madansky, 1959-1). In this chapter

we shall examine some of the solved problems in this area, cautioning the reader that the treatment is incomplete and that much research remains to be done."

Agricultural economists have contributed significantly to the treatment of risk in programming. In particular, McFarquhar[27] has shown that the risks associated with British arable farming can be satisfactorily treated.

2.2.4.3 Quadratic Programming

Beale[6], in outlining the history of non-linear programming at a symposium held in Cannes in the summer of 1964, showed that the first venture into the theory of non-linear programming has been to the problem known as quadratic programming. Quadratic programming is the name given to the special problem of minimizing a convex quadratic objective function of variables subject to linear restraints. This new extension to linear programming (Wolfe 1959, Beale 1959) may have a very considerable potential in that it broadens the field of application of linear programming. The method developed by Wolfe[43] is a variation of the standard simplex method. Beale[6] has developed a method bearing his name. Both methods appear to be practicable for use by those who are familiar with the general application of linear programming.

The references listed to this point have been concerned with the application and extension of linear programming as a means of solving problems. There are, of course, shortcomings to the method and these should be considered particularly in assessing program results*. Edwards[15] lists four areas of shortcomings, a) mathematical, b) statistical, c) economic, and d) programmers. The knowledge of the existence and extent of these shortcomings constitutes a considerable spur to continued development and extension of linear programming.

* Clark and Simpson have warned of the gap between theory and practice.

3. INVESTIGATIONAL

The investigation took the form of collecting, assessing and preparing data to meet the requirements for farm planning, using the technique of linear programming, together with the preparation and solution of a series of linear programs for a representative group of arable farms. Farm visits were made to each co-operating farmer. The farming operation was discussed in detail in the physical and financial terms most familiar to the farmer. A linear program was prepared for each farm, representing the farm as faithfully as possible and providing for a range of additional farming activities. Data from which the coefficient for the program were developed were obtained from many sources. A listing of the sources together with the assessment and preparation of these data constitutes much of the text of this section. This material on data collection and preparation is presented in two parts, the first dealing with data for the variable cost portion of the farm enterprise and the second dealing with the development of equations for some normally-fixed-cost entries so as to permit their inclusion as variable-cost entries in linear programming.

3.1 VARIABLE-COST DATA PRESENTATION

3.1.1 INTRODUCTION

Linear programming permits the analysis of any function in the program as an implicit function of the whole program. If the full potential of this powerful analytical device is to be realized, however, all functions making up the program must be as carefully prepared as the function developed for detailed investigation. Whereas the principle objective of this investigation was to study a particular function in the normally-fixed-cost section, it is obvious that equal care had to be taken with the variable-cost section. This requirement justified a careful analysis of data related to the variable-cost section, but in addition, this analysis constitutes a very useful exercise in itself. This is particularly so because whereas most work published on linear programming is concerned primarily with the variable-cost section, very little reference has been made to the agricultural data available from research and extension sources in Scotland.

3.1.2 LAND USE

Farmers usually express their restrictions on arable land use in one of two ways; they may express limits on particular crops as minimum or maximum percentages of total available acreages, or they may express limits in terms of rotations. Fig. 3.1 illustrates an example of the first method, while Fig. 3.2 illustrates the second.

Conditions: Grass not less than 1/4 of total.
Wheat not to exceed barley.
Potatoes equal to 1/6 of total.

RHS		Grass	Wheat	Barley	Potatoes
T	≥	1	1	1	1
T	≥	4			
0	≥		1	-1	
T	=				6

Fig. 3.1

Conditions: 3 year Grass not less than 1/4 of total.
Wheat not to exceed barley.
Potatoes equal to 1/6 of total.

RHS		PCCGGG	PCCCCC	W	B
T	≥	6	6		
0	≥	-1	1		
0	≥	-2	-5	1	1
0	≥			1	-1

Fig. 3.2

The method of Fig. 3.1, while apparently straight-forward, may not produce results that are easily translatable into practice. The method as illustrated in Fig. 3.2 produces results that are more readily translatable into practice, especially on farms with well established field boundaries. The maximum inconvenience in this example would be to build one dividing fence and remove another each year. Fig. 3.2 also illustrates, by means of the example of wheat and barley, the method by which any of the crops comprising the rotations may be brought out as separate columns. It may or may not be desirable to do this in any particular problem.

3.1.3 FERTILIZERS, NATURAL AND CHEMICAL

There are two distinct aspects to problems that involve fertilizers. The first is the representation of the generally non-linear response curves of crops to fertilizer application. The second is the problem of substitution rates between farm yard manure and chemical fertilizers.

The generation of linear approximations to observed crop response to fertilizer application has been widely treated[24]. The method of linear approximation generally followed is developed in section 3.2.5.

The response of crops to fertilizer becomes of significance in the preparation of linear programs primarily when working capital is a limiting resource. Under other conditions, optimum fertilizer applications should be used. As working capital was not found to be a limiting resource for any of the programs, fertilizers appeared only as variable cost items in preparing gross margins and as N, P, K and fibre requirements where substitution is possible.

Farmers in Scotland frequently list application of farm yard manure as a requirement for certain crops, particularly potatoes. They usually express their application of manure in tons per acre. Literature shows little consistency to the term, "ton of manure", either as related to the production of manure, or to its chemical composition. Hall[21], while measuring application of farm yard manure in tons per acre, and providing tables of representation composition based upon a ton, measures manure production in terms of the N, P, K and fibre in the feedingstuffs and litter provided. He mentions only casually manure production in tons per animal. Furthermore, figures relating feedingstuffs and litter to tons of manure produced, show a wide range of values. This is, of course, to be expected because of the range of moisture content that might be experienced. For these reasons, it becomes more practical to base application on the same basis as production, i.e.: N, P, K and fibre available, as related to feedingstuffs and litter.

Farm records provide a means of correlating N, P, K, and fibre application to a farmer's estimate of application in terms of tons per acre. For example, a record of animal numbers on farm L-3 indicated that an estimate of 12 tons per acre was in fact the manure produced by 3 bullocks over a 200-day feeding period. Feeding records showed that the average feed and litter supplies per bullock was as shown in Fig. 3.3 from which the N, P, K and fibre can be estimated.

	N	P_2O5	K_2O	Fibre
	lb.	lb.	lb.	Tons
½ ton barley	7.4	5.6	5.0	.21
3 tons silage	10.4	4.7	22.3	.34
1½ tons straw (litter)	6.1	5.4	21.5	.63
Total	23.9	15.7	48.8	1.2

Fig. 3.3

Estimated composition of manure from one animal, fed as shown.

Fig. 3.3 is based upon plant food values and proportions of plant food that are present in the dung from "Residual Values of Fertilizers and Feeding Stuffs"[19]. For example, this publication lists barley as having the following plant food percentages:

N	1.65
P_2O_5	0.75
K_2O	0.55

Percentages of plant foods that are present in the dung are listed as:

N	40
P_2O_5	67
K_2O	80

from which the plant food contained in the dung produced by feeding one ton of barley can be calculated as:

$$N = 1.65/100 \times 40/100 \times 2240 = 14.8 \text{ lb.}$$
$$P_2O_5 = 0.75/100 \times 67/100 \times 2240 = 11.2 \text{ lb.}$$
$$K_2O = 0.55/100 \times 80/100 \times 2240 = 9.86 \text{ lb.}$$

Figures for the percentage of fibre present in the dung are listed by Hall[21] as between 40 and 60 per cent of the original dry matter, depending upon exposure to elements, duration of storage and degree of decomposition. A figure of 50% was suggested by Smith[48], and was used in Fig. 3.3.

Fig. 3.4 illustrates a linear programming format that permits a prescribed requirement for manure to be met from plant food found in manure, or alternatively, from chemical fertilizers and crop residue returned directly to the land, or from a combination of the two.

			Crops to be fertilized (Acres)	Crop residue (Tons)	Animal feed (Tons)	Chemical Fertilizers (Units) N	P	K	Commercial Fibre (Tons)
N	0	≥	87	-4.5	-14.8	-100			
P	0	≥	50	-3	-11.2		-100		
K	0	≥	20	-22	-9.86			-100	
Fibre	0	≥	2		-.5				-.5
Balance	0	≥	-1.25	1					
	Z	=		Cost of 2 cwt. of Sulphate of Ammonia					

Fig. 3.4

There is by no means complete agreement on the value of fibre returned through animal manure as compared to straw and residue plowed back, or as compared to the inclusion of a grass break in the rotation.

Hood and Procter[28] report that results from an intensive cereal growing experiment show cereal yields following one year leys were superior to those obtained following other one-year breaks, including rape grazed, mustard plowed, and fallow and continuous cereal. They report also that burning straw and plowing straw back had no significant effect on yield, (provided 0.4 cwt. of N_2 was supplied per acre of straw plowed in) and that all treatments resulted in a reduction of soil organic matter.

Cook[15], reviewing results from experiments on the effect of herbage crops by crops that follow, concludes that the nitrogen supplied by herbage crops is the greatest factor affecting the yields of following crops. There were only occasionally recorded benefits from extra organic matter.

Smith[48] agrees that it is difficult to measure the influence of organic matter on crop yields. He suggests, however, that where a farmer has maintained or improved yields and feels that the fibre returned by farm yard manure is a factor, quantitative data for the substitution of straw plowed in for fibre from dung is necessary. He says that straw, in decaying after being plowed in, goes through some of the same chemical reactions as straw tramped in dung, in particular there is a loss of dry matter, and nitrogen is required for the reaction. He suggests that dry matter loss is about 50% and that 2 cwt. sulphate of ammonia per ton of straw should be provided. These figures have been incorporated in Fig. 3.4 and in linear programs L-3 and L-6. In Fig. 3.4 the required nitrogen is costed in the cost row, and in program L-3 an alternative approach is used. The nitrogen for fixation (requirement) is handled by a requirement placed in the straw column and the nitrogen balance row.

All of these reports would seem to verify the value of including N, P and K balances in linear programs that include cropping, but they do little to support the inclusion of a fibre balance. However, where farmers wish to change practice without changing the amount of fibre returned to the land, the format as developed in Fig. 3.4 should ensure that their wishes are met.

3.1.4 RATION FORMULATION

Linear programming has been used extensively for least-cost ration formulation[24]. Fig. 3.5 illustrates a format for compounding a ration from a variety of ingredients to a particular specification.

FEED-MIX PROBLEM*

We consider the problem of a feed manufacturer who has a proprietary brand of feed which is warranted to contain at least 35 per cent (by weight) protein, at least 1.5 per cent fat, and not more than 8 per cent fiber. He wishes to find the minimum cost of ingredients for one ton of feed. The manufacturer can purchase four primary feeds: alfalfa meal at \$66 per ton, distillers solubles at \$92 per ton, fish meal at \$156 per ton, and soybean meal at \$96 per ton. The percentages of fiber, protein, and fat of the different nutrient sources are given in table 4.3. These percentages serve as our "input-output coefficients" for the cost minimizing problem.

* Copied from reference 24.

Basic Data for Feed-Mix Example

Nutrient	Unit	Requirement	Alternative nutrient sources and amount of nutrient provided by each			
			Alfalfa meal	Distillers solubles	Fish meal	Soybean meal
		B	P_1	P_2	P_3	P_4
Fiber	%	not more than 8	25	3	1	6.5
Protein	%	at least 35	17	25	60	45
Fat	%	at least 1.5	2	5	7	.5
Bulk	ton	equals 1	1	1	1	1
Net cost $ per ton			66	92	156	96

The data in the columns indicate the percentage composition of the sources. Alfalfa meal has 25 per cent fiber, 17 per cent protein, and 2 per cent fat. The figures for fiber, protein, and fat in table 4.3 are expressed in percentages, whereas bulk and cost are expressed in tons. We could convert the percentage figures to a per ton basis so that the fiber content of alfalfa meal would be .25 tons, the protein content of fish meal would be .60 tons, and the fat content of soybean meal would be .005 tons. Since there is no need to make this change of units, we will continue to work in percentage terms.

The manufacturer's objective is to find the ingredients which will meet his warranty at minimum cost.

Fig. 3.5

It is entirely reasonable to conclude that the flexibility of an overall program for a farm in which livestock may constitute a significant enterprise, can be increased by permitting the livestock ration to be selected from among possible home grown crops and purchased feedingstuffs in a least cost formulation. (It should be noted that "cost" here becomes "opportunity cost".) By substituting an animal nutritional requirement column for the specification in Fig. 3.5, and by including home grown feeds, least-cost ration formulation can be included in the preparation of an overall farm program.

There are several quantitative approaches to ration formulations. Current British practice is to follow the Starch Equivalent[51] method. North American practice is largely based upon the total Digestive Nutrient Method[35]. Recent work by Blaxter[8,9] has resulted in another basis for formulation, at this date referred to as the "New Method".

All of these methods yield relationships that permit their inclusion in linear programs.

3.1.4.1 The Starch Equivalent Method

Table 3.1 shows the composition of foods commonly fed in the Lothians. Table 3.2 shows a summary of livestock requirements. These tables were prepared from standard sources[51] modified[20] by results from continuing research.

Composition of Foods

	S.E.	D.C.P. (%)	O.M. digy. (%)	D.M. (%)
Swedes	7	0.6	90	10
Sugar beet tops	9	2.0	85	17
Barley	70	7.0	80	85
Barley straw	20	0.8	40	85
Silage	11	1.8	70	22
Hay	27	3.8	55	85
Soya bean meal	64	40.4	85	89
Potatoes	19	1.1	90	24
Kale (green crop)	10	1.5		

Table 3.1

Category of Animal	Description	S.E. lb./day	D.C.P. lb./day	Max. dry matter lb./day for digestibility ranges 50-60	60-70	70-80
1.	Suckler cow, calved in the autumn. To be fed in court until pasture available in spring	11.3	1.48	17.3	20.3	33.4
2.	5 cwt. steer calf brought into court in October. To be fattened to market weight					
a.	at 1.8 lb. gain/day	8.0	0.94	10.9	12.7	14.8
b.	at 2.3 lb. gain/day	8.9	1.05	"	"	"
3.	Same as 2. except $4 - 4\frac{1}{2}$ cwt. heifer					
a.	at 1.6 lb. gain/day	7.3	0.83	10.5	12.2	14.1
b.	at 2.0 lb. gain/day	8.2	0.97	"	"	"
4.	4 cwt. steer calf maintained in store condition over winter	6.0	0.82	8.9	10.5	12.1
5.	$8\frac{1}{2}$ - 9 cwt. steer. Fattened in court at $1\frac{1}{2}$ lb. gain/day	8.5	0.82	12.9	15.1	17.5

Table 3.2

The calculation of the requirements as shown in Table 3.2 are contained in Appendix A.

Examinations of Tables 3.1 and 3.2 show that three requirements must be met. The requirements for Starch Equivalent and protein can be met directly by linear substitution and, therefore, present no difficulty.

Fig. 3.6 illustrates the format required.

Requirement			Bullock (5 cwt.) for 210 days at 1.8 lb. gain/day	Silage (Tons)	Barley (Tons)	Soyabean Meal (cwt.)	Sugar Beet Tops (Tons)
S.E. balance	0	$\geq$	1680	-266	-1568	-71.7	-202
Protein balance	0	$\geq$	197	-40.3	-157	-45.2	-44.8

Fig. 3.6

Columns 4, 5 and 6 of Table 3.2 indicate that dry matter intake is limited by the digestibility of a feedingstuff. Even a cursory examination of Table 3.1 will show that an animal on a high plane of nutrition cannot meet nutritional requirements from say, barley straw alone, because of limitations on dry matter intake.

It becomes highly desirable, therefore, that dry matter intake vs. digestibility-range data should be incorporated as a requirement in linear programs for livestock rations.

From the relationships between maximum dry matter intake and digestibility ranges of Table 3.2 a mathematical relationship can be expressed as follows:

$$DM_{max} = ADI \times D$$

where DM_{max} = maximum daily dry matter intake in pounds

D = digestibility range, expressed as a decimal

ADI = absolute daily intake

from which

$$ADI = \frac{DM_{max}}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

Substitutions of values from Table 3.2 into Equation 3.1 show ADI to be a constant for each class of livestock over the range of digestibility listed.

Values for ADI as calculated are listed in Table 3.3

Livestock	ADI
Suckler cow	31.4
8-3/4 cwt. steer	23.4
5 cwt. steer	19.8
4-1/4 cwt. heifer	19.1
4 cwt. steer	16.2

Table 3.3

Note: It has been pointed out[9] that this linearity is not maintained for very low digestibility ranges.

Dry matter supplied to an animal by any one feed may be written as:

$$DM = W \times C_{DM}$$

From 3.1,

$$W \times C_{DM} \leq ADI \times D$$

or rewriting,

$$W \times \frac{C_{DM}}{D} \leq ADI \quad \ldots\ldots\ldots\ldots\ldots \text{(3.2)}$$

Taking the extreme case, i.e.: where

$$W \times \frac{C_{DM}}{D} = ADI$$

and remembering that a condition of linearity is that, if

$$\frac{C_{DM}}{D} \; W = ADI$$

then

$$\sum_{i=1}^{n} \frac{C_{DM}}{D}_{i} \lambda_{i} W_{i} = ADI \quad \text{.............} \quad (3.3)$$

where

$$\sum_{i=1}^{n} \lambda_{i} = 1.$$

Noting that $\lambda_i W_i$ represents the amount of a particular feedingstuff fed, and reverting to the normal case, i.e.: an inequality, the following equation can be written:

$$\left(\frac{C_{DM}}{D}_{\text{feed}_1} \times \begin{matrix}\text{Weight}\\ \text{of}\\ \text{feed}_1\end{matrix}\right) + \left(\frac{C_{DM}}{D}_{\text{feed}_2} \times \begin{matrix}\text{Weight}\\ \text{of}\\ \text{feed}_2\end{matrix}\right) + \left(\frac{C_{DM}}{D}_{\text{feed}_n} \times \begin{matrix}\text{Weight}\\ \text{of}\\ \text{feed}_n\end{matrix}\right)$$

$$\leq ADI_{\begin{matrix}\text{particular}\\ \text{class of}\\ \text{animal}\end{matrix}} \quad \text{.......} \quad 3.4$$

Noting that this expression is of the same form as those for Starch Equivalent and Protein, it can be incorporated into a linear program format. The coefficients C_{DM} and D from Table 3.1 are combined in Table 3.4 to give a "fill coefficient" per pound, $\frac{C_{DM}}{D}$, for each feedingstuff.

Feedingstuff	Fill Coefficient, per pound $\frac{C_{DM}}{D}$
Swedes	0.111
Sugar beet tops	0.200
Barley	1.06
Barley straw	2.13
Silage	0.314
Hay	1.54
Soyabean meal	1.05
Potatoes	-
Kale	-

Table 3.4

Fig. 3.7 adds the dry matter intake limitation to the Starch Equivalent and Protein balances as shown in Fig. 3.6 by the addition of row three

Requirement			Bullock (5 cwt.) fed for 210 days at 1.8 lb. gain/day	Silage (Tons)	Barley (Tons)	Soyabean Meal (cwt.)	Sugar Beet Tops (Tons)
S.E. balance	0	≥	1680	-266	-1568	-71.7	-212
Protein balance	0	≥	197	-40.3	-157	-45.2	-44.8
Dry matter limit	0	≥	-19.8 × 210	0.314 × 2240	1.06 × 2240	1.05 × 112	0.200 × 2240
Beet top limit	0	≥	-1				2

Fig. 3.7

It can be seen that all of the requirements and limitations imposed by the data from Tables 3.1 and 3.2 can be incorporated into a linear program by means of three rows, if a combined ration for all livestock is required, or by three rows per class of livestock, if a breakdown of rations per class is required. (It is quite possible that many farmers would prefer the former. This permits the farmer to exercise his preferences in deciding how the sum of the feedingstuffs should be divided.)

Restrictions not appearing in Table 3.2 are usually related to maximum intake of particular feeds, e.g., sugar beet tops and potatoes. Restrictions of this nature can be readily handled in the manner illustrated by line four, Fig. 3.7, where the sugar beet top intake is restricted to one-half ton per bullock. (This figure would be arrived at by combining a top daily limit with the period in which beet tops would be available.)

3.1.4.2 The "New Method" For Feeding Standards

Calorimetric research carried out by Blaxter[8,9] and associates at the Hannah Dairy Institute on ruminant nutrition, has led to Blaxter's proposing a new, rational feeding system.

Blaxter presents evidence of shortcomings in both the starch equivalent and the TDN systems. Much closer agreement with actual performance is claimed* for the new system, as compared to either the starch equivalent or the TDN system.

It is not proposed to enter into discussion on the merits of the three systems. It will be shown, however, that the "new system" may be adapted to the technique of linear programming by introducing an approximation**.

Blaxter develops the following equations for the joint solution to growth and fattening problems.

* Prediction of performance for the "New Method", ± 0.2 lb./day as against ± 0.5 lb./day for the Starch Equivalent method.

** This approximation leads to an error that, over a range of tests with oats and hay did not exceed 0.25%.

Metabolizable energy of ration

$$= M \text{ (kcal)}$$

$$= \sum_{i=1}^{n} M_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.5)$$

where

M_i = metabolizable energy of one ingredient in ration

Dry matter in ration

$$= D \text{ (lb)}$$

$$= \sum_{i=1}^{n} D_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.6)$$

where

D_i = weight of dry matter of one ingredient in ration.

Concentration is defined as:

$$\text{Concentration} = \frac{M}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.7)$$

True metabolizable energy

$$= M^{l} = \frac{1}{f} \times M \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.8)$$

where

$$f = 1.00 + 0.11 \; L - \frac{190 \Delta L}{M/D} \quad \ldots\ldots\ldots\ldots \quad (3.9)$$

$$\Delta L = L - 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.10)$$

L = feeding level expressed as a multiple of the maintenance requirement (e.g.: maintenance, 1.5 x maintenance, etc.)

Metabolizable energy for maintenance

$$= \frac{1}{k_m} \times \text{fasting metabolism} \quad \ldots\ldots\ldots\ldots\ldots \quad (3.11)$$

where

$$k_m\ (\%) = 54.3 + 0.0143 \quad \frac{M}{D} \quad \ldots\ldots\ldots\ldots\ldots \quad (3.12)$$

Metabolizable energy for production

$$= \frac{1}{k_p} \times \text{energy for fat deposition} \quad \ldots\ldots\ldots \quad (3.13)$$

where

$$k_p\ (\%) = 3 + 0.405 \quad \frac{M}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.14)$$

Blaxter's equations may be arranged to permit a direct solution to problems that can be defined as follows:

1. Fasting metabolism and energy requirements for fat deposition known.

2. Metabolizable energy and dry matter content per pound of ration is known.

The following sequence of calculations may be used:

1. Calculate $\frac{M}{D} = \frac{m}{d}$

where

m = metabolizable energy, kcal/lb

d = dry matter, lb/lb

2. $k_m = 54.3 + 0.0143 \frac{M}{D}$ (Eq. 3.12)

3. $k_p = 3 + 0.0405 \frac{M}{D}$ (Eq. 3.14)

4. Energy for maintenance $= M_1 = \frac{\text{Fasting Metabolism}}{k_m}$ (Eq. 3.11)

5. Energy for production $= M_2 = \frac{\text{Energy for fat deposition}}{k_p}$ (Eq. 3.13)

6. Metabolizable energy $= M = M_1 + M_2$

7. True metabolizable energy $= M' = \frac{M}{f} =$

$$\frac{M}{1 + .11\left(\frac{M'}{M_1} - 1\right) - \frac{190}{\frac{M}{D}}\left(\frac{M'}{M_1} - 1\right)} \qquad \text{(Eqs. 3.8 and 3.9)}$$

It may be observed that the equation in step 7 is a quadratic of the form $aM'^2 + bM' + c = 0$

The roots of the equation are:

$$M' = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

8. Weight of ration, lb, $= \frac{M'}{m}$

A further step yields the feeding value of the ration directly in terms of fasting metabolism and energy for fat deposition.

$$\text{Feeding value} = V = \frac{\text{fasting metabolism + energy for fat deposition}}{W}$$

where

V = feeding value kcal/lb

It may be observed that for a particular ration, the feeding value remains constant for all cases where

$$\frac{\text{fasting metabolism + energy for fat deposition}}{\text{fasting metabolism}} = P = \text{a constant}$$

From this identity, tables of feeding values (kcal/lb) can be developed for feedingstuffs in terms of the parameter P. This permits the feeding requirement to be expressed as:

$$WV = P \times (\text{fasting metabolism}) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.15)$$

Feeding values, V, can be calculated for individual feeds, as well as for combined rations. If this is done, the following approximation may be made:

$$\sum_{i=1}^{h} W_i \; V_i = P \times (\text{fasting metabolism}) \quad \ldots\ldots \quad (3.16)$$

The error introduced by the use of this approximation to compound rations arises in that the concentration $\frac{M}{D}$ (appearing inherently) is weighted by <u>fractional proportions</u> of the ration, rather than by the <u>proportion by weight</u> (avoirdupois) i.e. there is no error when $\frac{M}{D}$ values are equal.

A series of calculations* using oats and hay from proportions of 0/100 to 100/0 and for P = 1, 1.5 and 2 indicates the error in using equation (3.16) to be less than 0.25%. If this error is representative over the whole range of normal feedingstuffs, then equation (3.16) can be used with confidence for the solution to practical problems.

Equation (3.16) permits the "new method" to be programmed as simply as can the SE method. Fig. 3.8 illustrates linear program entries for one animal that has a fasting metabolism of 6000 kcal and a requirement for fat deposition of 3000 kcal, and another that has a fasting metabolism of 7000 kcal and is to be fed at maintenance level. Oats and hay are permitted as feeds. Table 3.6 was prepared from Appendix B.

* Calculations shown in Appendix B.

Class of Animal	Fasting Metabolism kcal/day
A	6000
B	7000

Table 3.5

P	1.0	1.25	1.50	1.75	2.0
Feed	Feeding Value, kcal/lb				
Oats	825		727		674
Hay	625		501		432

Table 3.6

		Animal A at P = 1.5	Animal B at P = 1.0	Oats to Animal A lb./day	Oats to Animal B lb./day	Hay to Animal A lb./day	Hay to Animal B lb./day
0	$\geq$	9000		-727		-501	
0	$\geq$		7000		-825		-625
Z	=			-4.2d	-4.2d	-3d	-3d

Fig. 3.8

3.1.4.3 Grazing Animals

Continuing research, such as is being conducted by Bianca and Blaxter[7] et al on environmental effects, and work of grazing, should permit the calculation of energy requirements for grazing livestock. Continuing research such as is being carried out by J.C. Holmes[26], W. Holmes[27] and Cunningham[17] on herbage growth and yield, should permit the calculation of the energy produced by herbage under grazing. It should then be possi'le to equate energy required to energy produced. At this point in time, however, research has not been carried far enough to predict performance of grazing livestock on this basis. It becomes necessary to continue to estimate the performance of grazing livestock on the basis of grazing trials.

It is well known that the instantaneous yield of herbage is not a constant function of time. Contributing factors as listed by W. Holmes in influencing the growth pattern under grazing are climatic, seasonal and nutritional in addition to the pattern of grazing. J.C. Holmes shows that, although the production curve may be modified, it cannot be made to coincide with the herbage requirement for a uniform stocking rate, either on a year round basis, or through the normal growing season.

Grazing experiments have shown[36] that highest beef production per acre does not coincide with highest rate of gain per animal. These experiments have also shown that it is not possible to obtain maximum production per acre by continuous grazing. A rotational practice must be followed to prevent the land being bared, with a consequent rapid drop in grass production.

For these reasons, the mathematical representation of grazing livestock at a high level of management, must be based upon a tested grazing system.

The grazing system used for the linear programs included in this thesis was proposed by J.C. Holmes[26]. The system permits a high rate of gain per animal, while maintaining a good per-acre production. This system is based upon grazing trials of bullocks grazing a three year grass clover sward coupled with a one year ley of Italian Rye Grass.

The system provides a combination of rotational grazing over four fields and the clipping of excess seasonal production for silage and hay, to maintain a balance between production and requirements. Details of the rotation are shown in Appendix C.

It must be possible to describe systems for grazing management in terms of linear functions of production and consumption, and linear restraints, if these systems are to be included in a linear program. The grazing system as proposed by Holmes can be entered in its entirety in a linear program, but care must be taken that all aspects of the system are accounted for. Fig. 3.9 illustrates a typical approach to entering this system in a linear program.

Requirement			GGGCCP	CCGCCP	3 year grass (acres)	Rye grass (acres)	$8\frac{1}{2}$ cwt. bullocks	Hay (tons)	Silage (tons)
3 year grass balance	0	≥	-3		1				
Rye grass balance	0	≥	-1			1			
Grazing balance (3 year)	0	≥			-1		3/4		
Grazing balance (rye grass)	0	≥				-1	1/4		

Fig. 3.9

Where animals other than $8\frac{1}{2}$ cwt. fattening bullocks are entered in programs shown in this thesis, adjustment of acreage required for grazing was made as a linear function of starch equivalent feeding requirements.

Upward adjustment of stocking rate was made in some cases on the basis of individual farm records of grazing.

3.1.5 LABOUR

It is normal linear programming practice to enter available hours of labour in the restraint column on a seasonal basis, and to enter seasonal job requirements in the matrix. In order to accurately represent the actual labour position on a farm, it is necessary that a linear programming presentation permit a considerable degree of flexibility in labour availability. Farmers resort to many strategies. Non-weather-dependent work that can be put aside is undertaken during periods of inclement weather. Overtime may be employed. Extra labour may be hired on a short-term basis. Farming activities may be pushed back beyond the normally scheduled time for completion. It is necessary that all of these possibilities be recognized in the preparation of a linear program. No difficulties arise in providing for these possibilities; however, the cost-row coefficients may be difficult to assess. There is no well-established market for skilled short-term farm help. The economic penalties incurred by pushing operations back are difficult to estimate due to the variability of weather. Monte Carlo techniques show promise[31,52] in estimating weather effects by simulation. No exhaustive development has, however, been undertaken along these lines in either the Lothians or in Alberta.

Fig. 3.10 shows a partial linear program in which all of the normally employed strategies are entered. It may be observed that, in this form, whereas labour becomes increasingly expensive, it never becomes a rigid restraint.

			Enterprise 1	Enterprise 2	Sept. Overtime	Borrowed from October	Sept. Casual
Sept. Labour	500	≥	20	15	-1	-1	-1
Sept. O.T.	250	≥			1		
Oct. Labour	500	≥				1	
	Z	=	GM_1	GM_2	O.T. Wage	Penalty	Cas. Wage

Fig. 3.10

For the preparation of the farm programs, "arbitrary" values were developed for the relationships between weather-dependent and total available seasonal labour, and between labour available for overtime and weather-dependent activities. Data published in the Climatalogical Atlas of the British Isles[18] were used in the calculation of these arbitrary values. It was estimated that a rainfall in excess of 0.4 inches would stop field work for two days. In addition, the presence of fog was considered to stop harvesting. Hours of available overtime were restricted to 50% of the hours of regular time available for weather-dependent activities.

The existence of agricultural meteorological data relating common weather parameters to farming operations would be of very great assistance in refining work scheduling. Unfortunately, this aspect of agricultural meteorology has not been developed in the areas studied.

3.1.5.1 Job Requirements

With some notable exceptions (e.g. milking routines), work study has not been employed to a very large extent to provide detailed standard time data of a comparable nature to that provided for industrial activities. Where studies have been carried out, work elements are usually large, (e.g. plowing an acre of ground), and a detailed description of the job and the working place is usually not provided. (Mechanized operations are usually easier to describe, and so time studies may show pertinent data such as forward speed.) In general, it is particularly difficult to confidently transpose data prepared for one geographic area, to another. These factors prompted the preparation of time data for farm operations in the Lothians from a series of economic surveys carried out for the area by the Department of Economics, Edinburgh School of Agriculture[32, 33, 41, 42, 45]. These surveys were not intended primarily to provide detailed time data. However, the times required to perform the operations associated with the particular enterprise under study were recorded. Operational times obtained from this source are average times, and do not take into account the influence of machine size. These times should be, however, representative of present practice in the area, as a high degree of uniformity in machine size was noted between the individual returns from which the survey averages were obtained. Tables D1 - D4, Appendix D, are a compilation of time data from this source. Tables D5 - D9 comprise time data from other sources[5,6,10,11,34,38,39] in Great Britain. Tables D10 - D12 present data on livestock feeding, haying and silage making developed from studies on Alberta farms carried out by the Department of Agricultural Engineering, University of Alberta. The elements from which time is recorded in Tables D10 - D12 are generally much shorter than in Tables D1 - D9. Care must be exercised in their use to ensure that all of the work elements that are required to make up an operation are in fact included in the calculation of a standard time. Tables D10 - D12 approach the concept of standard data as developed for industrial operations. There should be no geographic limitation to their use.

3.1.5.2 Crew Size

The entire farm plant, including the element of crew size, is treated as a fixed-cost resource in Section 3.1. For the preparation of conventional programs, i.e., programs in which the farm plant is treated as a fixed-cost portion of the farm business, this requirement is normally pre-judged, and the regular labour force established accordingly. In the preparation of the linear programs for the Lothian farms, each farmer was asked to clear this aspect of the program by an expression of opinion as to whether or not he could cope with the enterprises entered.

3.1.6 COST-ROW COEFFICIENTS

Greatest flexibility is achieved when enterprises with saleable end-products are built up as combinations of variable activities. Under these conditions few column variables have true gross margins as cost-row coefficients. True gross margins are rather summations of gross returns and partial costs, usually involving several columns. Standard references for use with the gross margin technique for farm planning usually present gross margin figures on a per-acre basis. These figures must be broken down if they are to be used as cost-row coefficients for most linear programs. Appendix E, prepared by J. Harkins, University of Edinburgh, illustrates the calculation of partial gross margins for livestock. The true gross margin can be determined only after a solution to the program has been obtained.

Cost-row coefficients for crops were calculated on a basis of variable costs only where alternative opportunities existed (e.g., barley), and on a basis of gross return less variable costs where no alternative opportunity to direct sale existed (e.g., wheat and potatoes).

3.1.7 SUMMARY

Research in the application of linear programming to farming practice has clearly established the validity of utilizing linear programming as a method of selecting variable-cost enterprise combinations. The validity of a linear program depends upon the reliability of data available from scientific agricultural research and farming practice and the ability to express this data in a form suitable for programming. The value of rotations in maintaining and improving land fertility is not clearly established. However, as many farmers wish to maintain rotations, and as fixed field boundaries limit cropping flexibility, some method of respecting these limitations must be imposed. Very often entering two rotations, e.g., GGGCCP and CC break CCP and permitting a continuously variable relationship between them will tidily meet all restrictions and limitations.

It is much more convenient and more accurate to assess the manurial value of dung in relationship to the feedingstuffs consumed than on a per-animal basis. This basis permits substitution between tramped dung, sludge and commercial N, P and K on a direct basis. Any of three methods of ration formulation (SE, TDN, and New Method), may be included in a general linear program to provide a least-cost ration, simultaneously with the selection of an enterprise combination. (Cost, as used here, implies opportunity cost.) Three equations are required, one an energy balance, one a protein balance, and the third a digestibility-fill relationship. All the necessary relationships are substantially linear and may be developed from existing data. In particular, the "New Method", as developed by Dr. K.L. Blaxter, appears to present no difficulties in its application to ration formulations in the general context of a farm linear program. The higher correlations between observed and predicted animal performance claimed for this method provide a strong argument that this method should be developed for widespread use.

Grazing animals are normally treated in association with pasture to form an implicit system of management. No problems arise in maintaining this concept of an implicit system, provided that the necessary factor-factor relationships between the rotation and the grazing animals are entered.

There are twin labour conditions, availability and job requirements. Availability for particular operations is a function of the variable, weather. Data relating farm operations to normal weather (i.e.: rainfall, wind, RH and cloud cover) are not available. These relationships could preferably be developed in a comparatively short period of time. Once available, the massed weather data, presently couched in terms of conventional parameters, would become useful for a statistical approach to the effect of weather on farm operations.

Such normal farm strategies as the use of overtime, short-term help, and pushing operations back, can be readily provided for in linear programming.

Time data for farming activities are generally not as precise as for industrial activities. Coarse time data collected in one area are likely to be unreliable if used in another. The

Appendix includes time data for field operations in the Lothians prepared from economic surveys. Standard times for livestock feeding activities attained from stop-watch studies in Alberta are also included.

3.2 FIXED-COST DATA PRESENTATION

3.2.1 INTRODUCTION

It has been shown in Section 2 that it is convenient for the purpose of linear programming to divide enterprise costs into variable and fixed-cost categories. Fixed-costs normally enter into the calculation of profit, but do not influence the selection or scale of enterprises beyond providing restraints.

This treatment of fixed costs adequately represents such factors as rent and rates, but it does not fairly represent the position of machinery and buildings in farm planning. Many management decisions involve the substitution of fixed-cost items for variable-cost items (for example, machinery in place of casual labour). To achieve the highly desirable objective of having important decisions based upon "opportunity costs", it must be possible to permit the substitution of normally fixed-cost items, for normally variable-cost items (and vice-versa), and to select lowest cost combinations of fixed-cost items.

3.2.2 CAPITAL COST CALCULATIONS

The validity of much of what will be developed in Section 3.2 depends upon the ability to be able to express the yearly capital cost of farm structures and equipment as a percentage of the new, or replacement cost. Numerous text books and technical papers deal with this subject[13,37,50]. In general, a yearly capital cost assessment depends upon a forecast of the useful life of the asset; an estimate of its disposal value; the effect of provisions for tax relief; and the assessment of an interest charge on the invested capital.

This relationship is expressed mathematically by Eq. (3.17).

$$C' = \frac{\int^{n} \begin{matrix}\text{yearly depreciation}\\ \text{and interest charges}\end{matrix}\, dt \;-\; \begin{matrix}\text{provision for}\\ \text{tax relief}\end{matrix}}{n} \quad \ldots\ldots (3.17)$$

Eq. (3.17) in the integral form. It is often expressed as

$$C' = \frac{C}{n'} \quad \text{..............................} \quad (3.17a)$$

where

n' = pay-off period, which can be calculated after solving for C' in Eq. (3.17) as

$$n' = \frac{C}{C'}$$

Interest charges should normally not be entered in the calculation of capital costs that are to appear in linear programs. The fundamental basis establishing the validity of a linear programming solution is that each enterprise appearing in the solution justifies its presence in terms of its marginal value product, i.e.: its value as measured in terms of alternate courses of action. In other words, the program solution itself establishes the interest rate.

Exceptions to this rule are called for under some circumstances. The use of borrowed capital is one; the lack of provision in the program for an alternative investment opportunity, such as the purchase of bonds, is another.

Eq. (3.17) yields a constant value for the estimated yearly capital cost, c'. This does not, however, imply a straight-line depreciation curve. A depreciation curve simply provides an estimate of residual value. It is entirely appropriate to distribute the cumulative depreciation uniformly over the period of utilization, regardless of the relationship between yearly increments of depreciation.

Programming, as with other decision-making aids, involves a choice from among alternatives. It follows that it must be possible to equate like parameters of each alternative, including cost-row coefficients. As decisions involving fixed assets are influenced by the trade-in values of assets that could be replaced, it is necessary to include the influence of trade-in values in cost-row coefficients. Cost-row coefficients for fixed assets (as will be developed) are normally functions of yearly capital cost. It is, therefore, convenient to include the effect of trade-in in yearly capital-cost calculations. This may be accomplished by adding a term to Eq. (3.17) to yield Eq. (3.18).

$$C' = \frac{\int^{n}\begin{pmatrix}\text{yearly depreci-}\\\text{ation and}\\\text{interest charges}\end{pmatrix}dt - \begin{pmatrix}\text{provision}\\\text{for tax}\\\text{relief}\end{pmatrix} + \begin{pmatrix}\text{unrealized capital}\\\text{value and interest}\\\text{on asset being}\\\text{replaced}\end{pmatrix}}{n} \quad \text{.. (3.18)}$$

As with Eq. (3.17), Eq. (3.18) may be written as:

$$C' = \frac{C}{n'} \quad \text{.............................. (3.18a)}$$

The illustration of a simple budget as a means of selecting between the retention of an existing asset, and its replacement by an alternative, affords a means of examining Eq. (3.18). Assume that the variable costs are equal, and that the alternative has become available before the expiration of the useful period of life originally estimated for the existing asset.

The yearly capital cost of the asset on hand, as calculated by Eq. (3.18) remains precisely the same as the calculation made at the time of its purchase, provided that the course of action implied by its retention is based upon continued use through the useful life-span, originally assumed. The yearly capital cost of the alternative, as calculated by Eq. (3.18) includes the present actual disposal value of the asset on hand. The actual disposal value (as distinct from some previous estimate of future value), becomes very important in determining the yearly capital cost of the alternative, and through this, the future course of action.

It should be observed that this calculation involves only two imponderables -- the disposal value of the existing asset and the disposal value of the alternative. All other figures are exact. The use of Eq. (3.18) thus affords a sound accounting procedure with a minimum reliance on estimates of future value.

3.2.3 CLASSIFICATION OF FIXED-COST ITEMS

The Production Handbook gives the following definition:

> "The term 'factory' or more broadly, 'manufacturing plant', means a building or group of buildings together with mechanical equipment, machinery, tools and various other physical facilities needed for the production of goods together with the physical,

mental, and in the broader sense, the sound well-being of the employees."

Related to farming, this sentence could be re-phrased to read:

"The term 'farm production plant' means land and buildings, together with mechanical equipment, machinery, ------------------------."

In attempting to simultaneously plan both the farm production plant and a farm enterprise combination, it is necessary to develop characteristic equations for the production plant.

The replacement costs of buildings, machines and other items of production equipment are usually functions of the size of the enterprises with which the structures or machines are associated. Expressed mathematically,

$$C = f(S) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.19)$$

where

C = replacement cost

S = enterprise size

Enterprise size may in some cases be expressed as a function of a rate of performance, or throughput and time.

$$S = g(Y,t)$$

where

Y = a rate of performance

t = time

Eq. (3.19) may be written as,

$$C = h(Y,t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.20)$$

Farm structures are usually associated with segments of farm enterprises in which size is not well defined by the flow parameters Y and t. Eq. (3.19) in one independent variable S usually best defines the relationship between replacement cost and enterprise size for farm structures. On the other hand, the size of most mechanized operations is well defined by throughput and time. Eq. (3.20) best defines the general relationship between replacement cost and enterprise size for farm machines.

It has been shown in Section 3.2.1 that yearly fixed-cost may be represented as a function of replacement cost. Eqs. (3.19) and (3.20) may be rewritten as Eqs. (319a) and (3.20a) to yield the more useful relationship between yearly fixed-cost and enterprise size.

$$C' = u(S) \qquad \qquad (3.19a)$$

$$C' = v(Y, t) \qquad \qquad (3.20a)$$

Particular solutions to the general relationships expressed by Eqs. (3.19 - 3.20a) can be developed for a number of farm fixed-cost problems. These particular solutions can be classified by characteristic equations, or by problem definition. It is felt that it is more useful for the purpose of this thesis to classify solutions by the second method. Problems are defined as relating to:

1. Farm structures
2. Materials handling
3. Field machines
4. Crew size

These divisions, while lacking precise definition, permit familiar examples to be borne in mind while developing characteristic equations.

3.2.4 BUILDINGS AND STRUCTURES

Buildings are normally required to provide a particular environment for humans or animals, a working area, or a storage space. The characteristic dimensions of size are usually floor area and volume. As floor area and volume normally bear a particular relationship to the unit of measure of the associated enterprise, it is convenient to express building requirements as linear coefficients of enterprise size, e.g., square feet per animal, cubic feet per crop acre, etc.

Eq. (3.19) can be rewritten as:

$$C' = \frac{C}{n'} = \beta RD$$

yielding

$$\beta = \frac{C}{RDn'} \qquad \qquad (3.21)$$

where

β = fixed cost per year per unit of enterprise

C = new cost of structure

R = units of enterprise per unit of building dimension

D = characteristic dimension (ft^2, ft^3)

n = pay-off period

For a rectangular livestock-housing building with sides a and b and height h, Eq. (3.21) yields

$$\beta = \frac{K_1 + K_2 + 2K_3 h\left(\frac{a+b}{ab}\right)}{Rn'}$$

where

β = yearly fixed cost per animal

K_1, K_2 and K_3 are unit building costs of floor, roof and walls respectively

The β vs. Animals relationship is plotted in Fig. 3.11 for

a conventional general-purpose structure with concrete floor and truss supported roof.

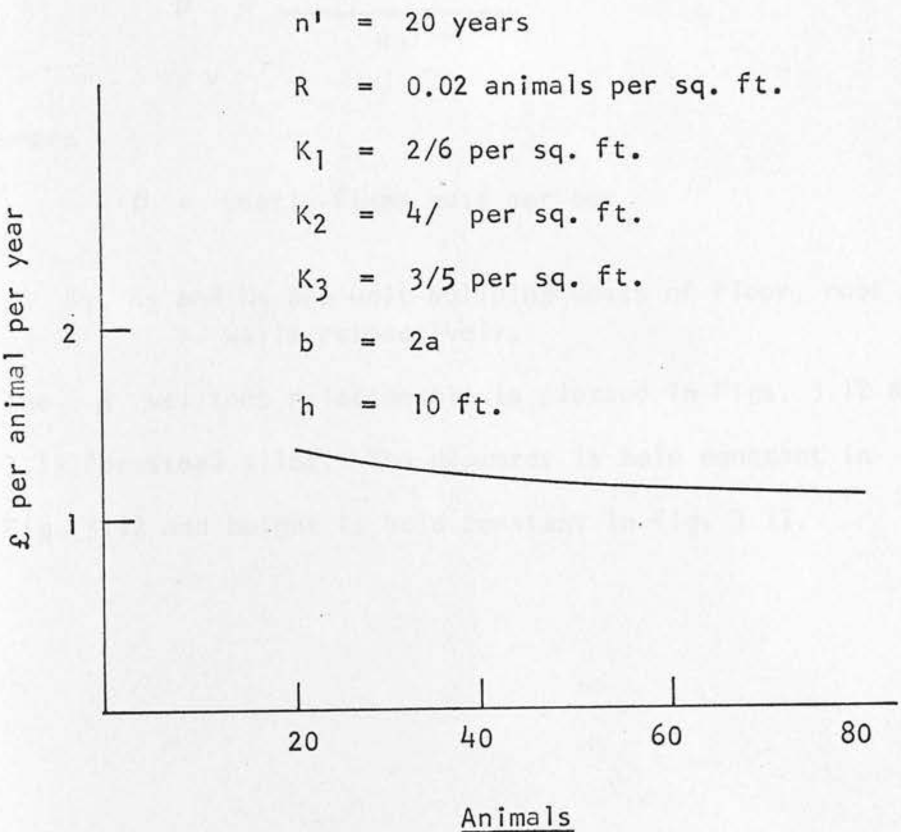
n' = 20 years
R = 0.02 animals per sq. ft.
K1 = 2/6 per sq. ft.
K2 = 4/ per sq. ft.
K3 = 3/5 per sq. ft.
b = 2a
h = 10 ft.
£ per animal per year
2
1
20
40
60
80
Animals

Fig. 3.11

It may be observed that a linear approximation does not introduce any serious error over the useful size range of the building.

For a circular silo of diameter d and height h, Eq. (3.21) yields:

$$\beta_v = \frac{\dfrac{K_1 + K_2}{h} + \dfrac{4K_3}{d}}{Rn'}$$

where

β = yearly fixed cost per ton

K_1, K_2 and K_3 are unit building costs of floor, roof and walls respectively.

The β vs. tons relationship is plotted in Figs. 3.12 and 3.13 for steel silos. The diameter is held constant in Fig. 3.12 and height is held constant in Fig. 3.13.

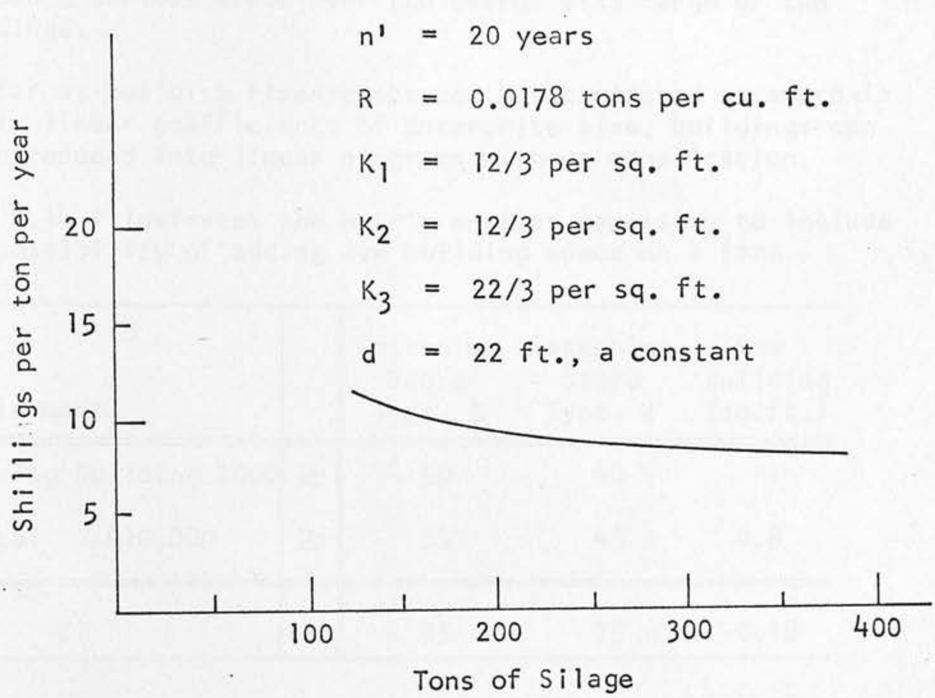

Fig. 3.12

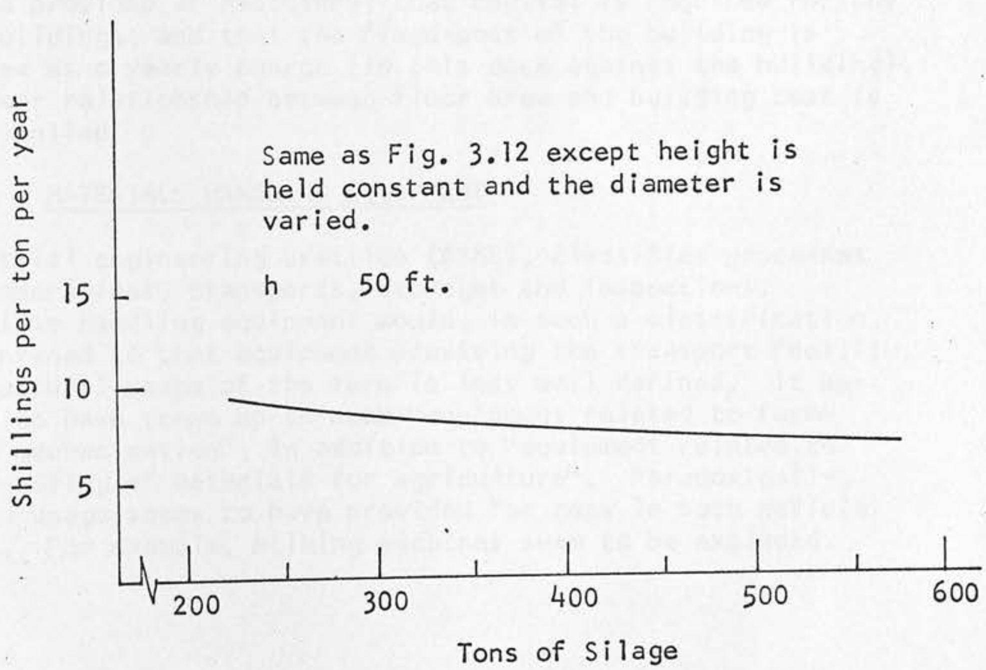

Fig. 3.13

It may be observed that, again, linear approximations do not introduce serious error over the useful size range of the buildings.

Insofar as building fixed-costs can be considered as approximately linear coefficients of enterprise size, buildings can be introduced into linear programs without complication.

Fig. 3.14 illustrates the matrix entries necessary to include the possibility of adding new building space on a farm

Requirement		Fattening Store Type 1	Fattening Store Type 2	New Building (sq.ft.)
Existing Building 2000	≥	50	40	-1
Capital £10,000	≥	55	45	0.8
Z	=	85	75	-0.18

Fig. 3.14

No significance should be attached to the particular values of the matrix entries. The figure illustrates that new building can be provided if necessary; that capital is required for any new buildings; and that the fixed-cost of the building is carried as a yearly charge (in this case against the building). A linear relationship between floor area and building cost is also implied.

3.2.5 MATERIALS HANDLING EQUIPMENT

Industrial engineering practice (ASME), classifies processes into operations, transports, storages and inspections. Materials handling equipment would, in such a classification, be confined to that equipment providing the transport facility. Agricultural usage of the term is less well defined. It appears to have grown up to mean "equipment related to farmstead mechanization", in addition to "equipment related to the handling of materials for agriculture". Paradoxically, common usage seems to have provided for gaps in both definitions. For example, milking machines seem to be excluded.

For the purpose of this thesis it is convenient to lump a wide range of farmstead equipment together, and to classify this as materials handling equipment. Augers, Elevators, Hoppers, Mixers, Blenders, Silo Loaders and Unloaders, Unloading Wagons and Blowers can be included among other items in this classification.

Characteristics of Materials Handling Equipment

Recently, a great deal of emphasis has been placed on the use of mechanical equipment to lighten the choring task. This emphasis led to the appearance of a profusion of new items of equipment on the market. There has not, as yet, been time for design to harden along particular lines, with a wide range of sizes in each line.

In the present period of rapid development, combinations of materials handling equipment exhibit the characteristic of having a discontinuous capacity spectrum. An additional characteristic is the availability of a variety of dissimilar combinations that will perform the same task. For the purpose of this thesis, these are the characteristics identified by the designation: "Materials Handling Systems". Future developments in materials handling equipment will likely resolve along the same lines as have developments in harvesting and threshing equipment. Components will be factory coupled to reduce a sequence of operations to a single mechanized task. At such time as this occurs, the designating will lose its significance. For the present, however, it is useful to consider the designation "Materials Handling Equipment" as a division in the range of fixed-cost items.

Consideration of the mechanics of creating any one of the possible permutations that could be identified as a materials handling system is not included here. At present, a promising approach to this micro-aspect of machinery selection would appear to be through an extension of the use of the principles underlying Critical Path Scheduling and allied techniques. Integer programming has also been used[45].

The characteristic equations for materials handling systems are:

$$T = \frac{A}{K} \quad \text{..........................} \quad (3.22)$$

and $C' =$ the yearly fixed-cost, a constant .... (3.23) where

A = size of task (or enterprise)

K = system capacity, a constant

T = limiting time

It is normal programming practice to enter Eq. (3.22) with K as an input-output coefficient as a means of relating enterprise size to available time through machine capacity. When used for this purpose, a cost-row coefficient is not required, and the fixed-cost of the machine does not enter as a factor in selecting the enterprise.

There are two very good reasons why cost-row coefficients for materials handling equipment should be included in linear programs.

1. The fixed-cost of the equipment might influence the selection and scope of the associated enterprise.
2. The inclusion of a range of alternative systems, each offering a different rate of work, or capacity, removes the time factor as a rigid restraint in determining enterprise scale.

The influence of machine fixed-cost can be provided for by adding a machine component to the appropriate column cost-row coefficient.

From Eq. (3.23) it is apparent that this component is non-linear and the contribution to the value of the functional takes the form:

$$\Delta Z = \frac{C'}{T} T \quad \text{or} \quad \Delta Z = \frac{C'}{A} A \quad \text{......} \quad (3.24)$$

Where a range of alternatives are permitted, equations (3.22) and (3.24) may be rewritten as

$$T = \sum_{i=1}^{n} \frac{A_i}{K_i} \quad \text{........................} \quad (3.25)$$

$$\sum_{i=1}^{n} \Delta Z = \sum_{i=1}^{n} \frac{M_i}{A_i} A_i \quad \ldots\ldots\ldots\ldots \quad (3.26)$$

Either of these pairs of equations may be programmed by entering the qualifying equations (3.22) or (3.25), as input-output coefficients, and by the cost-coefficients of Eqs. (3.24) and (3.26) to the constant cost-row coefficients* of A_i.

$$Z = \sum_{i=1}^{n} \left(GM + \frac{C'_i}{A_i} \right) A_i$$

The cost row now becomes non-linear, and some non-linear techniques must be employed to obtain a solution.

As mentioned in Section 2, Beale[4] and others have developed methods of representing non-linear cost-row coefficients as quadratic equations, and then by the use of modified Simplex, and other techniques, have developed solutions to this class of non-linear programming problems.

It has also been shown[29] that analog computers can, with very few additions to the normal linear-programming hook-up, provide solutions to non-linear problems. The pair of equations (3.25) and (3.26) have characteristics that have enabled integer programming to be used[45] in the selection of system components.

Examination of the non-linear cost-row coefficients of Eqs. (3.24) and (3.26) suggests that because of their special (and uniform) nature, it might not be necessary to employ general non-linear techniques to obtain satisfactory solutions to this class of problems.

A simple approach to the solution of problems represented by Eqs. (3.22) and (3.24) is to "guess" at the final value of the column variable, and then to calculate a constant cost-

* Normally the gross margin

coefficient such that multiplication by the column variable will yield the true yearly fixed cost. Armstrong and Faris[1] have reported that this method usually "zeros-in" on the second iteration.

Comparatively simple methods may also be used for solving problems that are represented by Eqs. (3.25) and (3.26).

Examination of the non-linear component of the cost-row coefficients reveals that these components approach infinity as the A_i approach 0, and that discontinuities exist in the regions of $A_i = 0$. Reference to the real problem reveals, however, that at $A = 0$, the fixed-cost of equipment <u>for that process</u> should also be zero. Minima may occur when any of the A_i are non zero, and the remaining A_i are zero. Any one of these local minima may be the true minimum, depending upon the alternate opportunities for T. It can be further rationalized* that no more than n local minima can occur provided that only one restraint is involved, or n(n-1) if there are two restraints.

One solution to problems of this nature is to solve the program n or n(n-1) times, entering only one alternative each run. The machine component of the cost-row can be treated using Eq. (3.24) as a cost-row component or it can be left out altogether and then net, rather than gross margins, compared.

Another approach is to guess at all relevant A values simultaneously and then proceed iteratively as outlined for Eqs. (3.22) and (3.24).

Writers[4,29] have pointed out that iterative methods may lead to local minimum values (solutions). The possibility of this presenting a problem with non-linear coefficients of the form illustrated in Eqs. (3.24) and (3.26) has not been explored.

A further approach may be provided in some cases by treating the alternatives as if they constituted a continuous curvilinear function, and then utilizing a linear approximation to

* One of the theorems underlying numerical solutions to m x n matrices is that there can be no more than m non-zero vectors.

this function to obtain a solution*. For example, if systems are available with fixed increasing capacities K_1 to K_n, the following qualifying expression relating task size to available time may be written:

$$T = \frac{A_1}{K_1} = \frac{A_2}{K_2} = \cdots\cdots \frac{A_n}{K_n} \qquad \cdots\cdots\cdots\cdots\cdots (3.27)$$

The following equations may be developed from the definition of the problem.

$$A_n = (A_1 - 0) + (A_2 - A_1) + \cdots\cdots (A_n - A_{n-1})$$

or

$$A_n = \Delta_1 A + \Delta_2 A + \cdots\cdots \Delta_n A$$

Similarly,

$$K_n = \Delta_1 K + \Delta_2 K + \cdots\cdots \Delta_n K$$

Therefore,

$$\frac{A_n}{K_n} = \frac{\Delta_1 A + \Delta_2 A + \cdots\cdots \Delta_n A}{\Delta_1 K + \Delta_2 K + \cdots\cdots \Delta_n K} \qquad \cdots\cdots (3.28)$$

Noting that,

$$\text{if} \quad \frac{A_2}{K_2} = \frac{A_1}{K_1} = T$$

$$\text{then} \quad \frac{A_2 - A_1}{K_2 - K_1} = \frac{\Delta_2 A}{\Delta_2 K} = T$$

$$\text{or} \quad \frac{\Delta_i A}{\Delta_i K} = T \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (3.29)$$

Substituting for $\Delta_i A$ in Eq. (3.28) yields

$$\frac{A_n}{K_n} = \frac{T\Delta_1 K + T\Delta_2 K + \cdots\cdots T\Delta_n K}{\Delta_1 K + \Delta_2 K + \cdots\cdots \Delta_n K}$$

* This method was developed by Charnes[14] and applies to separable convex functionals.

or

$$A_n = T\Delta_1 K + T\Delta_2 K + \text{--------}\ T\Delta_n K \qquad \ldots (3.30)$$

If equipment fixed-costs are constants as defined in Eq. (3.23),

then

$$C'_n = (C'_1 - 0) + (C'_2 - C'_1) + \text{----}(C'_n - C'_{n-1})$$

or

$$C'_n = \Delta_1 C' + \Delta_2 C' + \text{-------}\ \Delta_n C'$$

If these costs are to be represented by a coefficient times a column variable, then the cost-coefficients are non-linear as shown in Eq. 3.31.

$$C'_n = \frac{\Delta_1 C'}{\Delta_1 A}(\Delta_1 A) + \frac{\Delta_2 C'}{\Delta_2 A}(\Delta_2 A) + \text{---}\frac{\Delta_n C'}{\Delta_n A}(\Delta_n A) \qquad \ldots (3.31)$$

Eqs. (3.30) and (3.31) are illustrated in matrix form in Fig. 3.15.

		T	A	$\Delta_1 A$	$\Delta_2 A$	$\Delta_n A$
0	$\geq$		1	-1	-1	-1
0	$\geq$	-1		$\frac{1}{\Delta_1 K}$		
0	$\geq$	-1			$\frac{1}{\Delta_2 K}$	
0	$\geq$	-1				$\frac{1}{\Delta_n K}$
Z	=			$\frac{\Delta_1 C'}{\Delta_1 A}$	$\frac{\Delta_2 C'}{\Delta_2 A}$	$\frac{\Delta_n C'}{\Delta_n A}$

Fig. 3.15

Linear approximations to the cost coefficients may be developed in precisely the same manner as illustrated for Eqs. (3.22) and (3.24). The program is solved directly by the simplex method. As pointed out by Charnes[14], the simplex algorithm insures that A_n is developed as:

$$A_n = \sum_{i=1}^{n} \Delta_i A$$

Provided, and only provided that the function is convex over the region of approximation, i.e.,

$$\frac{\Delta_n C'}{\Delta_n A} > \frac{\Delta_{n-1} C'}{\Delta_{n-1} A}$$

The computational methods as developed for the selection of materials handling systems apply to any range of fixed-cost assets where the capacities over the range cannot be reasonably treated as a continuous function. Conversely, ranges of materials handling systems where capacities over the range can be reasonably treated as a continuous function should be considered, for purposes of computation as field or series machines.

3.2.6 FIELD OR SERIES MACHINES -- ONE-MAN CREW

Field or series machines are defined for the purpose of this thesis as machines that are available in an infinitely variable size range. The relationship between task size and time then appears as:

$$A = TY \qquad \text{(3.32)}$$

where

A = task size

T = time

Y = capacity or rate of performance

The assumption of continuously variable size ranges, of course, represents an approximation of a discrete variable by a continuous variable. Errors introduced by this approximation are considered to be a penalty that can be justified in view of the more powerful mathematical tools that are made available.

3.2.6.1 Energy Relationships

Field machines must meet work quality standards, e.g., tillage machines must provide a tilth and must destroy growing weeds; seeding equipment must correctly meter and place seeds; harvesting machines must deliver all of the harvested crop, and in an undamaged condition. Acceptable quality standards are, in most cases, poorly defined and are difficult to assess quantitatively. Farmers and their advisors, however, are fairly definite as to what constitutes "satisfactory" and "unsatisfactory" performance. Within the qualitative definition of satisfactory work, machines may supply energy to a system, and work may be performed. This concept has been widened in recent years to include system control.

It is useful to recall the energy balance concept as applied to farm work, in order to more rationally estimate the operating cost of farm machines.

An energy balance may be written as:

$$\eta E = W \quad \text{-------}$$

where

E = energy supplied

η = efficiency of energy conversion

W = work accomplished

It may be recalled that power may be defined as the rate of doing work. Then,

$$\eta PT = W$$

For a particular quantity of work accomplished

$$\eta \, PT = \text{a constant}$$

Where η is a constant, P and T may vary inversely while

maintaining a constant product. The quantity of energy that must be supplied varies directly with the work accomplished, and is independent of the rate at which the work is performed. Within well defined limits[3,43] the quantity of fuel required to perform a particular operation remains the same whether a large tractor and machine combination perform the task or whether a small tractor and machine are used over a longer period of time. Fuel costs may, therefore, normally be considered as a linear function of task size alone.

3.2.6.2 Timing of Operations

Weather and seasonal effects combine with growth characteristics of crops to determine the general periods in which particular farm operations, or sequences of operations, are best performed. Fig. (3.16) shows the periods in which representative farming activities are normally carried out in the Lothians. This chart was prepared from the records, and experience of the six co-operating farmers.

A survey is presently being carried out by Blyth[11] on a much larger sample to determine in detail the timing and duration of representative farming operations in the Lothians.

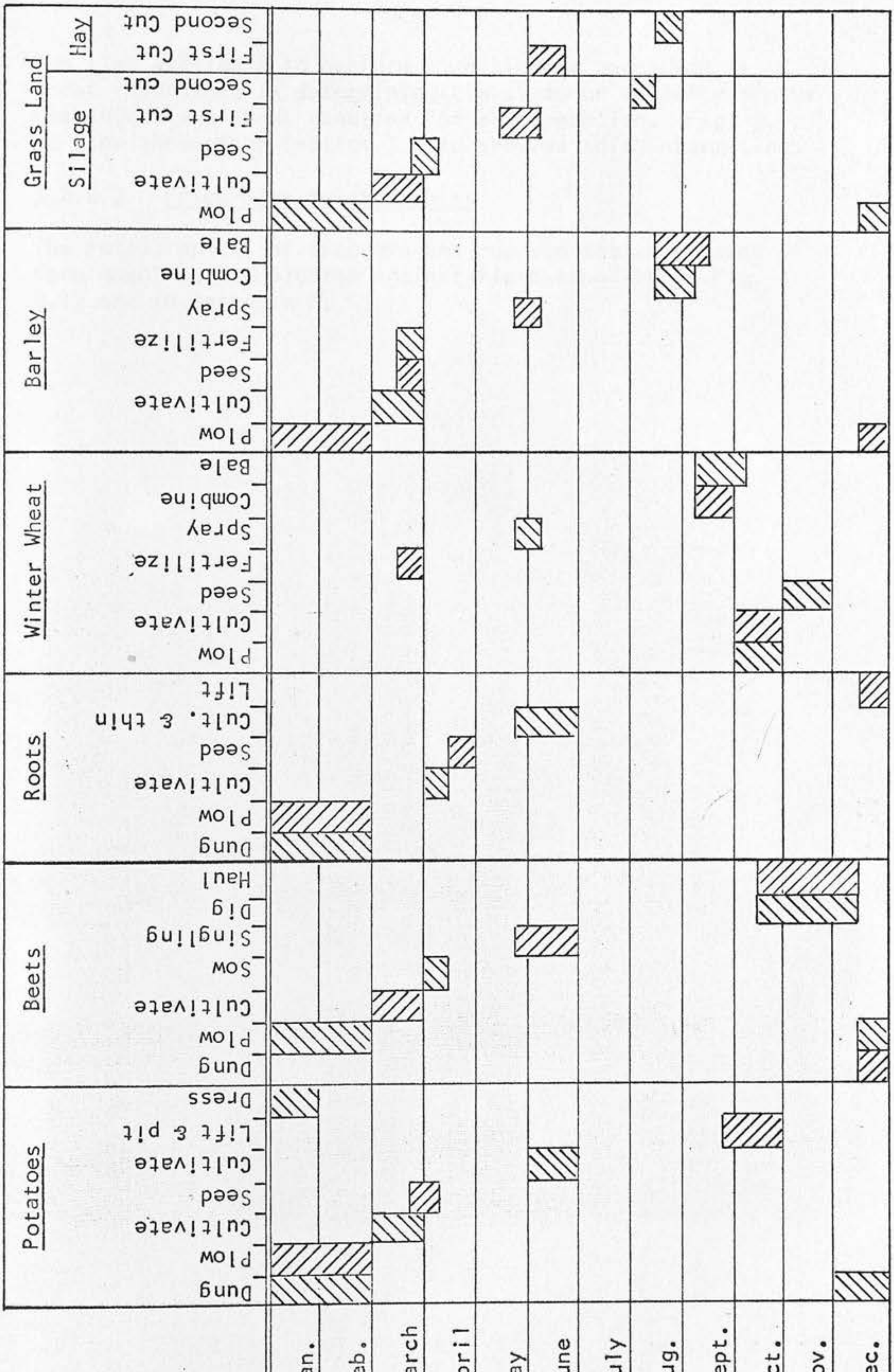
Potatoes
Dung
Plow
Cultivate
Seed
Cultivate
Lift & pit
Dress
Beets
Dung
Plow
Cultivate
Sow
Singling
Dig
Haul
Roots
Dung
Plow
Cultivate
Seed
Cult. & thin
Lift
Winter Wheat
Plow
Cultivate
Seed
Fertilize
Spray
Combine
Bale
Barley
Plow
Cultivate
Seed
Fertilize
Spray
Combine
Bale
Grass Land
Silage
Plow
Cultivate
Seed
First cut
Second cut
Hay
First Cut
Second Cut
Jan.
Feb.
March
April
May
June
July
Aug.
Sept.
Oct.
Nov.
Dec.

Fig. 3.16

the time available to perform a particular operation is of great importance in determining the size, or capacity of the mechanical equipment required for the operation. Fig. 3.16 is used throughout Section 3.2 to provide this information.

3.2.6.3 Price-Size Relationships

The retail prices of tractors and representative classes of farm machines are plotted against field capacity in Fig. 3.17 and in Appendix F.

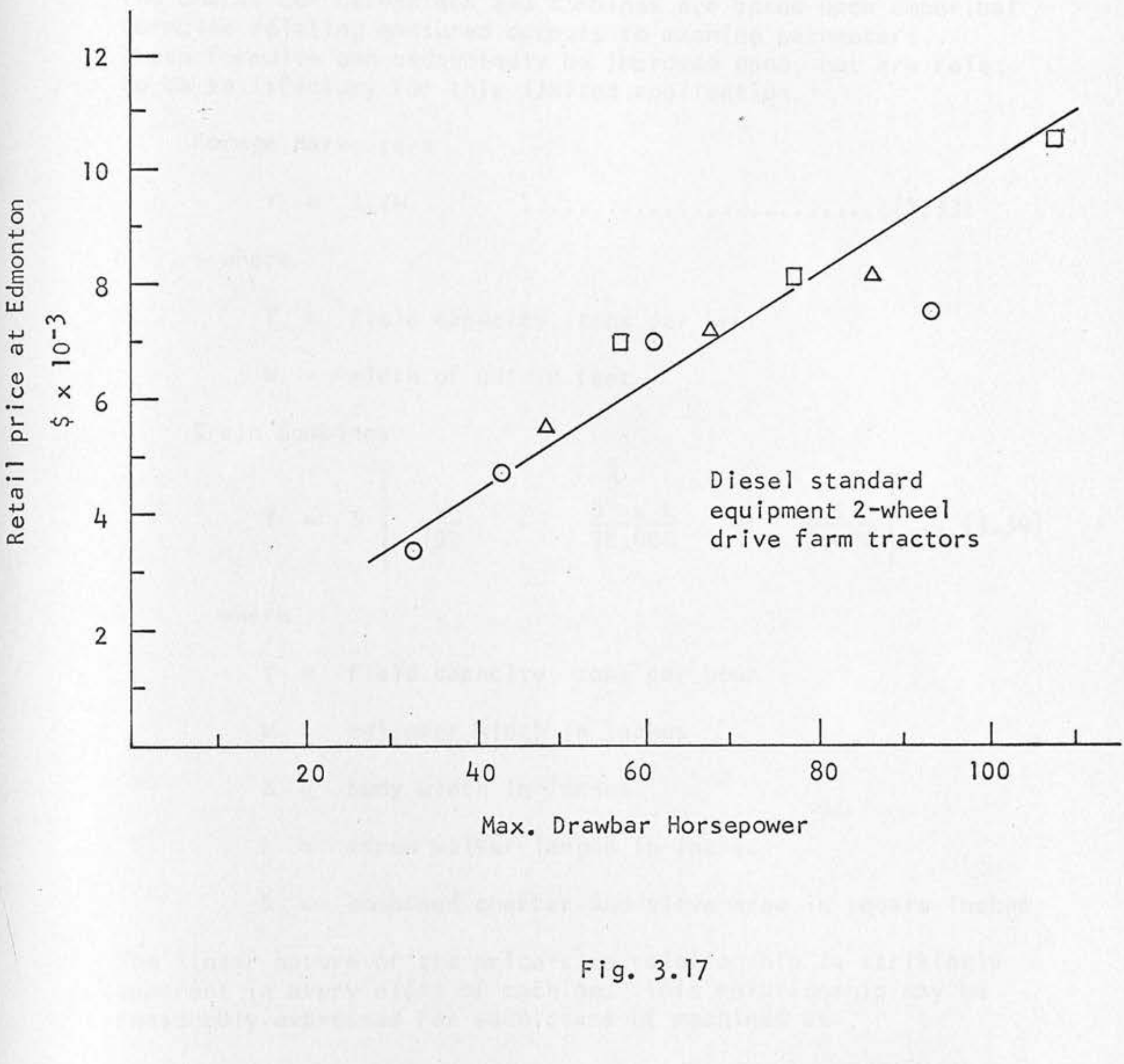
12
10
8
6
4
2
Retail price at Edmonton
$ × 10-3
20
40
60
80
100
Max. Drawbar Horsepower
Diesel standard
equipment 2-wheel
drive farm tractors

Fig. 3.17

The data for all of these charts, excepting those for the harvesters, are plotted in a straight-forward manner, and assume only that speeds are constant for a class of machines, and that field capacities are linear functions of operating width. The charts for harvesters and combines are based upon emperical formulae relating measured outputs to machine parameters. These formulae can undoubtedly be improved upon, but are felt to be satisfactory for this limited application.

Forage Harvesters

$$Y = 2.2W \qquad \dots\dots\dots\dots\dots\dots\dots\dots \quad (3.33)$$

where

Y = field capacity, tons per hour

W = width of cut in feet

Grain Combines

$$Y = 3\left\{\frac{W}{192} + \frac{B^{\frac{3}{2}} \times L}{38{,}600} + \frac{S}{7{,}400}\right\} \dots \quad (3.34)$$

where

Y = field capacity, tons per hour

W = cylinder width in inches

B = body width in inches

L = straw walker length in inches

S = combined chaffer and sieve area in square inches

The linear nature of the price-size relationship is strikingly apparent in every class of machine. This relationship may be reasonably expressed for each class of machines as

$$C = BY \qquad \dots\dots\dots\dots\dots\dots\dots\dots \quad (3.35)$$

where

C = retail price

Y = size (capacity) e.g., acres per hour

B = a constant of proportionality

It may be useful to express capacity in terms of average ground speed as

$$Y = Sy$$

where

S = forward speed (MPH)

y = capacity per MPH

The relationship expressed by Eq. (3.34) may be extended to relate capacity and yearly capital cost.

$$C' = B'Y \qquad \text{.................................} \quad (3.37)$$

where

C' = yearly capital cost

B' = a linear coefficient

C' may be calculated, as developed in Section 3.2.2. B' may then be calculated as

$$B' = \frac{C'}{C} \times B$$

The extremely useful relationship expressed by Eq. (3.37) is used repeatedly in Section 3.2.6 in developing equations for machinery selection and replacement.

3.2.6.4 <u>Programming for Equipment Selection</u>

It has been shown that the cost of energy is relatively independent of the rate at which work is performed, and that fuel costs may, therefore, normally be considered as constant coefficients of enterprise size. A similar case can be developed for repair and maintenance costs*. The more complex re-

* Tractor repair cost functions tend to be lumpy. However, the lumpiness is of a rather predictable nature. It then becomes entirely reasonable to prorate repair costs as linear functions of work accomplished.

lationships that exist between fixed-cost, machine capacity and task size are expressed by Eqs. (3.32) and (3.37).

$$A = YT \qquad \text{..................} \quad (3.32)$$

and

$$C' = B'Y \qquad \text{..................} \quad (3.37)$$

A working sequence of farming operations may be represented by Eqs. (3.32) and (3.37) as:

$$\sum A_i = Y_i t_i \qquad \text{..................} \quad (3.32a)$$

subject to the constraint

$$\sum_{i=1}^{n} t_i = T$$

and

$$C' = \sum_{i=1}^{n} B_i' Y_i \qquad \text{..................} \quad (3.37a)$$

minimize C'

Equs. (3.32a - 3.37a) are fundamental to programming for equipment selection.

Linear programming requires that at least one of the three terms of Eq. (3.32) or (3.32a) be a constant, and that B' in Eqs. (3.37) or (3.37a) be a constant coefficient of a variable Y. As A and T are normally system variables, the requirements for linear programming are not directly met.

A number of mathematicians have developed non-linear programming algorithms. The greatest emphasis has been placed on programs in which constraints are linear functions and cost coefficients are quadratic. Although it may be possible to rearrange Eqs. (3.32a) and (3.37a) to permit quadratic programming, it would appear to require an extension to present models to permit non-linear restraints. It was decided, rather than to attempt to program for equipment selection by extending non-linear models, to attempt to combine two other minimizing methods. Lagrange's method and the simplex method for linear programming are combined in a "solve and verify" sequence.

Referring to Eqs. (3.32) and (3.37), assuming no alternate use of labour, it can be shown that Y (and therefore C') approaches a minimum value when T approaches a maximum permissible value. Where maximum permissible values for T are known, Eqs. (3.32) and (3.37) may be readily included in a general linear program by including Eq. (3.32) in the restraint matrix and Eq. (3.37) in the cost row. This is illustrated by Fig. 3.18.

		Wheat	Barley	Potatoes	Plow Size (Y)	
0	≥	-1			30	Times Available
0	≥		-1		50	for Plowing
0	≥			-1	40	
Z	=				-B'	

Fig. 3.18

The method illustrated in Fig. 3.18 can be extended to the general problem (as represented by Eqs. (3.32a) and (3.37a) provided that "best values" can be developed for the time input coefficients "t_i".

Eqs. (3.32a) and (3.37a) can be written for a program of farm operations performed by one man operating a series of machines in sequence as shown in Fig. 3.19.

		Y_1	Y_2	Y_3	--	Y_h
A_1	$\leq$	$t_{1,1}$				
A_2	$\leq$		$t_{2,2}$			
A_3	$\leq$			$t_{3,3}$	--	
A_m	$\leq$					$t_{m,n}$

$T_1 \geq t_{1,1} + \cdot\cdot\, t_{3,3} + \ldots\; t_{m,n}$ (Where T_1 total work hours for period one)

$A_{m\ 1}$	$\leq$	$t_{m+1,1}$			
$A_{m\ 2}$	$\leq$		$t_{m+2,2}$		
$A_{m\ 3}$	$\leq$			$t_{m+3,3}$	
$A_{m\ p}$	$\leq$				$t_{m+p,n}$

(Where T_2 total work hours in period two)

$T_2 \geq t_{m+1,1} + t_{m+2,2} + t_{m+3,3} + \ldots\; t_{m+p,n}$

Z	=	B'_1	B'_2	B'_3	--	B'_n

Select t_{ij} so that $\sum B_i Y_i$ minimum

Where: Y_1 = capacity of machine

A_i = task size

t_i = time for single operation

T_i = time limit for sequence of operations

B'_i = cost coefficient

Fig. 3.19

For example, Y_1 might represent plow capacity, Y_2 disc capacity, Y_3 harrow capacity and Y_n seeder capacity, all in acres per hour.

T_1 might represent the time limit on field operations for planting the barley crop. It should be noted that the general formulation permits any number of implements, and any number of qualifying equations.

(i.e.: for $T_i = \sum t_{ji}$, i can be any integer)

Therefore, if competition for time exists between enterprises (as distinct from operations of one enterprise), the equation illustrated in Fig. 3.19 can be extended to include this possibility.

It should be further noted that the power-unit (tractor) can be included in the set of equations as shown in Fig. 3.20.

		Y_1	Y_2	---	Y_n	Y_t
0	≥	K_1				-1
0	≥		K_2			-1
0	≥				K_n	-1
Z	=	B'_1	B'_2		B'_n	B'_t

Fig. 3.20

The addition of a power unit to a list of machines completes the model. This model faithfully represents the farm machinery requirements for a pattern of farming when A_i and T_i are known and where a single operator is involved. Conditions under which this model is valid are in fact those present on many family farms. In western Canada family farms constitute a significant percentage of the total and this classification is not restricted to small farming operations. In the Lothians this model is not representative of a majority of arable farms, but, nevertheless, it represents one extreme alternative to the existing pattern of mechanized operation. The real importance of this model, however, is that it provides a useful foundation for the analysis of any pattern of mechanized operations.

The set of equations represented by Figs. 3.19 and 3.20 is capable of a mathematical solution that yields a level value for the function. A solution* to equations of this nature was formulated by Lagrange, and his method of solution is referred to in mathematical texts[16,44,49] as the method of Lagrange's Multipliers.

Lagrange's Multipliers may be used to obtain a level value for a function of the form:

$$u = F \quad x,y,z,\text{---}t \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.38)$$

where the variables are connected by an arbitrary number of auxiliary equations of the form:

$$\begin{aligned} \Phi_1 \; x,y,z,\text{---}t &= 0 \\ \Phi_2 \; x,y,z,\text{---}t &= 0 \qquad \ldots\ldots\ldots\ldots\ldots \quad (3.39) \\ \Phi_3 \; x,y,z,\text{---}t &= 0 \end{aligned}$$

(It should be observed that Eq. (3.38) could represent the functional of the equation in Fig. 3.19 while Eq. (3.39) could represent the auxiliary equations in time and tractor size of Fig. 3.19 and Fig. 3.20 respectively.)

Lagrange's method consists of forming the function:

$$u = F + \lambda \Phi_1 + \mu \Phi_2 + \psi \Phi_3 \qquad \text{---}$$

where λ , μ and ψ are constants to which shall

* Another promising method for solution involves non-linear programming on analog computers. The required computer diagram is shown in Appendix G.

later be assigned suitable values and where u is a function of the independent variables (x ---- t). The partial derivatives in (x ---- t) of this function give the equations:

$$F_x \quad + \quad \lambda\,\Phi_{1_x} \quad + \quad \mu\,\Phi_{2_x} \quad + \quad \psi\,\Phi_{3_x} \quad = \quad 0 \quad \ldots (3.40)$$

$$F_t \quad + \quad \lambda\,\Phi_{1_t} \quad + \quad \mu\,\Phi_{2_t} \quad + \quad \psi\,\Phi_{3_t} \quad = \quad 0$$

The unknowns x ---- t, can now be determined from Eqs. (3.39) and (3.40).

The combination of Lagrange's method and linear programming provides a powerful approach to problems involving both variable-cost and normally-fixed-cost relationships. A sequence can be devised as follows:

1. Solve a linear program without the inclusion of machine size entries to obtain acreages, or guess at final acreages.

2. Using these acreages, solve for the minimum cost machinery combination, using Lagrange's method.

3. Using time values obtained from (2), enter the machine-size relationships in the program matrix.

4. Solve the program and compare the final acreages with the acreages obtained from (1).

These four steps constitute one iteration of a combined "solve and verify" program. If there is lack of agreement, a second iteration can be performed starting at step (2) and using acreages obtained from (4).

A solution obtained in this manner insures the achievement of a final program in which items of machinery and enterprises are selected in mutual association.

Application of a general method to a real problem will yield the solution. However, the effect of particular problem variables on the solution may be obscured by the interaction of several variables and by the complex nature of auxiliary relationships. Some useful relationships can more readily be revealed by examining a series of simple examples.

Example 3.1

It is required to perform a single sequence of operations on a particular field within a time of T hours. The fixed-cost of each machine may be represented by the relationship:

$$C' = B'Y$$

Determine the lowest fixed-cost machinery combination. Assume adequate tractor power is available.

The function may be written:

$$Z = \sum_{i=1}^{n} C' = \sum_{i=1}^{n} B_i'Y_i$$

The auxiliary equation may be written:

$$T = \sum_{i=1}^{n} \frac{A}{Y_i}$$

Noting that A is a constant,

$$\frac{T}{A} = \sum_{i=1}^{n} \frac{1}{Y_i}$$

or

$$\sum_{i=1}^{n} \frac{1}{Y} - \frac{T}{A} = 0 .$$

Applying Lagrange's method:

$$Z = \sum_{i=1}^{n} B_i' Y_i + \lambda \sum_{i=1}^{n} \frac{1}{Y_i} - \frac{T}{A}$$

Writing the partial derivative in Y_i

$$B_1' - \frac{1}{Y_1^2} \lambda = 0$$

$$B_2' - \frac{1}{Y_2^2} \lambda = 0$$

$$B_n' - \frac{1}{Y_n^2} \lambda = 0$$

from which

$$\sqrt{\lambda} = \frac{\sum_{i=1}^{n} \sqrt{B_i'}}{\frac{T}{A}}$$

Substituting for λ in the partial derivatives yields:

$$Y_j = \frac{\sum_{i=1}^{n} \sqrt{B_i'}}{\sqrt{B_j'}} \frac{A}{T} \quad \text{.......... Ans.}$$

where the subscript j designates a particular value for i.

Examination shows that, as might have been expected, each Y varies directly with A and inversely with T. Of greater interest is the relationship between each Y and the B'. The ratio of optimum machine size is obtained by dividing, in turn, the square root of each particular B' value into the sum of the square roots of the B' values. This simple relationship has wide application to problems of optimizing machinery combinations.

Example 3.2

A series of operations must be performed each day. Some of the operations occur every day, while some of the operations are seasonal. Determine a least-fixed-cost machinery combination, where the yearly fixed-cost for each machine may be represented as:

$$C' = B'Y$$

This problem may be treated without loss of generality by assuming two operations per day, one an every-day operation, and one a seasonal operation. Similarly, there is no loss of generality by selecting the units of the every-day operation and one other so that these two operations are sized to be numerically equal.

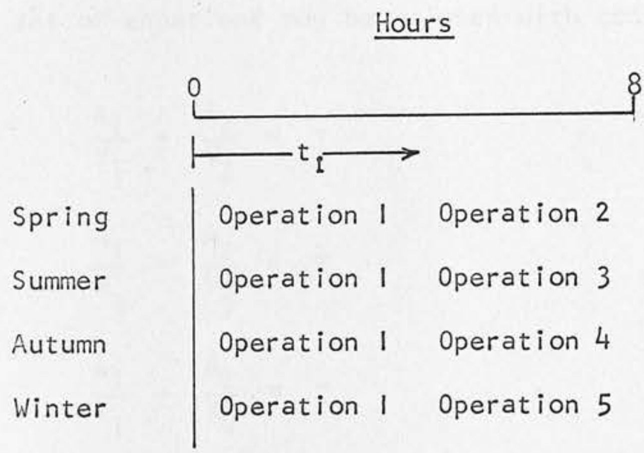

Fig. 3.21

If Operation 1 represents an operation of constant size, performed by a particular machine, then:

$$t_1 + t_2 = T$$

$$t_1 + t_3 = T$$

$$t_1 + t_4 = T$$

$$t_1 + t_5 = T$$

or,

$$t_2 = t_3 = t_4 = t_5$$

Suppose the size of the operation is A_1, ----- A_5 respectively.

then:

$$\frac{A_1}{Y_1} + \frac{A_2}{Y_2} = T$$

$$\frac{A_1}{Y_1} + \frac{A_3}{Y_3} = T$$

This set of equations may be written with constant numerators as:

$$\frac{A_1}{Y_1} + \frac{A_2}{Y_2} = T$$

$$\frac{A_1}{Y_1} + \frac{A_2}{Y'_3} = T$$

$$\frac{A_1}{Y_1} + \frac{A_2}{Y'_4} = T$$

$$\frac{A_1}{Y_1} + \frac{A_2}{Y_5} = T$$

It is apparent that $Y_2 = Y'_3 = Y'_4 = Y'_5$

Therefore,

$$Y'_3 = \frac{A_2}{A_3} Y_3$$

$$Y'_4 = \frac{A_2}{A_4} Y_4$$

$$Y'_5 = \frac{A_2}{A_5} Y_5$$

Writing an expression for combined capital cost,

$$C' = B'_1Y_1 + B'_2Y_2 + B'_3Y_3 + B'_4Y_4 + B'_5Y_5$$

Rewriting in terms of Y':

$$C' = B'_1Y_1 + B'_2Y_2 + \frac{A_3}{A_2} B'_3Y'_3 + \frac{A_4}{A_2} B'_4Y'_4 + \frac{A_5}{A_2} B'_5Y'_5$$

$$\text{or,}\quad C' = B'_1Y_1 + B'_2Y_2 + \frac{A_3}{A_2} B'_3Y_2 + \frac{A_4}{A_2} B'_4Y_2 + \frac{A_5}{A_2} B'_5Y_2$$

$$\text{or,}\quad C' = B'_1Y_1 + \left(B'_2 + \frac{A_3}{A_2} B'_3 + \frac{A_4}{A_2} B'_4 + \frac{A_5}{A_2} B'_5 \right) Y_2$$

This expression may be simplified as:

$$C' = B'_1Y_1 + B''_2Y_2$$

where

$$B'' = B'_2 + \frac{A_3}{A_2} B'_3 + \frac{A_4}{A_2} B'_4 + \frac{A_5}{A_2} B'_5$$

This equation represents a function to be minimized. A qualifying equation can be written as:

$$\frac{1}{Y_1} + \frac{1}{Y_2} = \frac{T}{A_1}^*$$

The function can be minimized by the method of Lagrange's multiples as in Example 3.1, yielding the solution:

$$Y_1 = \frac{\sqrt{B'_1} + \sqrt{B''_2}}{\sqrt{B'_1}} \quad \frac{A_1}{T}$$

Ans.

$$Y_2 = \frac{\sqrt{B'_1} + \sqrt{B''_2}}{\sqrt{B''_2}} \quad \frac{A_1}{T}$$

Y_3, Y_4 and Y_5 may now be calculated.

The optimum combination of sizes for machines used in daily sequence, one machine used every day and the remainder used less frequently, is seen to be effected by the frequency of machine use. As compared to non-seasonal combinations, machines that are used daily should be larger, while machines that are used less frequently should be smaller.

This relationship is of particular value in establishing the level of mechanization of farmstead operations (usually daily operations) in combination with field operations (usually seasonal in nature).

Example 3.3

Tow mechanized operations, each of a specific size, are to be performed in sequence within a specified time limit. Two similar mechanized operations, but of different specified sizes, are to be performed in sequence within a second specified time limit. Select the optimum combination of machine sizes. The Y for each class of machine is continuously variable, and the yearly fixed-cost may be represented as:

* A_1 and A_2 are assumed numerically equal.

$$C_i^! = B_i^! Y_i$$

The example may be illustrated by the following problem:

$$C' = 500\ Y_1 + 200\ Y_2$$

$$\frac{10}{Y_1} + \frac{20}{Y_2} \leq 10$$

$$\frac{20}{Y_1} + \frac{10}{Y_2} \leq 12$$

where the first expression is the function to be minimized, and the second and third are qualifying in-equations.

It is apparent that one of the inequations will in fact be an equality. Assume this to be the first. The second inequation can be made an equality by adding a dummy operation using a dummy machine with a capacity Y_3.

then,

$$\frac{10}{Y_1} + \frac{20}{Y_2} = 10$$

$$\frac{20}{Y_1} + \frac{10}{Y_2} + \frac{1}{Y_3} = 12$$

The function may be rewritten to include Y_3*, by placing a nominal value on the dummy operation:

$$C' = 500\ Y_1 + 200\ Y_2 + 1\ Y_3$$

Proceeding with Lagrange's method:

$$C' = 500\ Y_1 + 200\ Y_2 + 1\ Y_3 + \lambda \left(\frac{10}{Y_1} + \frac{20}{Y_2} - 10 \right)$$

$$+ \mu \left(\frac{20}{Y_1} + \frac{10}{Y_2} + \frac{1}{Y_3} - 12 \right)$$

* A condition for the application of Lagrange's method is that

λ and μ must not vanish.

The three partial derivatives may now be written:

$$500 - \frac{10}{Y_1^2}\lambda - \frac{20}{Y_1^2}\mu = 0$$

$$200 - \frac{20}{Y_2^2}\lambda - \frac{10}{Y_2^2}\mu = 0$$

$$1 - \frac{1}{Y_3^2}\mu = 0$$

The five equations in five unknowns (Y_1, Y_2, Y_3, λ and μ may now be solved to yield values:

$$Y_1 = 2.20$$

$$Y_2 = 3.66$$

$$Y_3 = 9.00$$

If only a single qualifying equation is involved, the solutions are:

For the first:

$$Y_1 = 1.89$$

$$Y_2 = 4.20$$

For the second:

$$Y_1 = 2.40$$

$$Y_2 = 2.69$$

The mathematics leading to the solution are sufficiently involved to prevent any generalization being made concerning the form of the solution. A comparison of the three sets of answers, however, emphasizes the over-riding importance of the B' values in determining a least-cost combination.

Example 3.4

Determine the optimum sizes of machines in a series to be used in sequence, the combined operations to be performed within a specific time. Include the tractor.

A combination of machine sizes that would demand a larger tractor than some other combination might, when included with the cost of the tractor, represent a higher combined yearly fixed-cost. Where this probability exists, the tractor should be included in arriving at the optimum combination.

The example may be illustrated by the following problem:

$$C' = 100\ Y_1 + 100\ Y_2 + 100\ Y_3$$

where

Y_1 = capacity (size of one machine)

Y_2 = capacity of the second machine

Y_3 = capacity (H.P.) of tractor

$$\frac{200}{Y_1} + \frac{200}{Y_2} = 100$$

$$Y_3 \geq 2\ Y_1$$

$$Y_3 \geq Y_2$$

The equation represents a time-size qualification, while the inequations represent the power requirements for the machines in the sequence.

Investigation of this problem reveals that three and only three possible conditions can apply: both restrictions imposed by power requirements can be equalities, or, one can be an equality while the other is an inequality. Solutions to all of these possible combinations are readily obtained.

Assume only one equality,

$$Y_3 = 2\ Y_1$$

Clearly the other power qualification cannot effect the solution if the solution obtained by neglecting it sustains the ignored qualification. By corolary, a solution can be obtained by ignoring the second qualification and checking the validity of the solution by determining whether or not the ignored qualification is sustained by the solution.

Ignoring the second power qualification and substituting for Y_3 in the functional, the problem appears as

$$Z = 300\, Y_1 + 100\, Y_2$$

$$\frac{200}{Y_1} + \frac{200}{Y_2} = 100$$

These equations solve readily by Lagrange's method to yield:

$$Y_1 = \frac{\sqrt{300} + \sqrt{100}}{\sqrt{300}} \times 2 = 3.15$$

$$Y_2 = \frac{\sqrt{300} + \sqrt{100}}{\sqrt{100}} \times 2 = 5.47$$

$$Y_3 = 6.30$$

and

$$Z = \$1,492.00.$$

The validity of the solution is proven by the second qualification being sustained by substitution.

$$Y_3 = 6.30 \qquad Y_1 = 5.47.$$

The solution to this example obtained by assuming the other qualifications to be an equality is of course, invalid by the corolary that has been established. The solution to this example obtained by assuming both qualifications to be equalities does not provide a minimum value for the function, yielding

$$Z = \$1,500.00.$$

It should be noticed that Lagrange's method is not required when all power qualifications are assumed to be equalities.

In summary, Lagrange's method has been shown to be capable of minimizing the summed fixed-costs of machinery combinations. In practice the method is extremely simple and unfailing in minimizing functions where only one qualifying equation exists, or where qualifying equations may be combined to yield a single equation. The method becomes generally unmanageable for people with limited proficiency in algebra as additional qualifying equations are added, because of the appearance of higher order roots. The special case presented by power qualifications partially yields to iterative procedures and is perfectly manageable where only a limited number of permutations occur.

3.2.7 FIELD OR SERIES MACHINES -- MULTIPLE-MAN CREW

The alternative to one-man operation of a farming unit is operation by a labour force that at times consists of two or more men.

The effects of additional farm labour on the economics of the farm can be measured in terms of the types of processes in which the additional farm workers are involved. It is convenient for this purpose to list three classifications of farm processes.

1. Processes consisting of a single operation.

2. Processes in which operations must be performed simultaneously by two or more workers.

3. Processes in which operations may be performed in sequence by one worker, or simultaneously by two or more workers.

3.2.7.1 Opportunity cost of additional labour for Processes consisting of a single operation

These processes may be represented by the equation:

$$A = YT$$

where

A = size of the process (operation)

Y = rate of performance, or throughput

T = time available

Where this type of process is performed by a machine and power unit having a fixed-cost characteristic of the form:

$$C' = B'Y$$

where

C' = yearly fixed-cost

B' = linear coefficient

Y = rate of performance or throughput

Additional labour has zero opportunity cost within the available size range of equipment.

Farming practice in the Great Plains areas of Canada and the United States bears this out. Grain farming in these areas consists of three seasonal processes: preparation of the seed-bed and seeding; summer-fallowing; and combine harvesting.

Developments over the years have tended towards making each of these processes a single operation. (Self-propelled combines, and combined seeding and tillage equipment can be cited.)

Paralleling these developments, machine (and tractor) sizes have continued to grow. Large crawlers and large 4WD wheel tractors such as the IHC 4300 and Wagner WA17 place about 200 drawbar horsepower under the command of a single operator. There is little reason to think that this trend to ever larger sizes will not be continued.

3.2.7.2 Opportunity Cost of Additional Labour for Processes in which Operations must be performed simultaneously by Two or More Workers

Processes of this type can be represented by the equation:

$$Y_1 = Y_2 \text{ ----- } Y_n = \frac{A}{T}$$

Insofar as this classification may represent a real situation, it is quite apparent that the opportunity cost of the required additional labour is equal to the opportunity cost of the enterprise with which the process requiring the extra labour is involved. Examples lending weight to this conclusion are many. Industrial assembly line processes are representative. The bargaining power of assembly line workers has traditionally been exercised in proportion to the profitability of the enterprise. The major factor in determining the number of regular workers employed on Lothian arable farms is probably the minimum number of regular workers that the farmers of the area feel are required for critical potato operations. The profitability of the potato enterprise justifies the retention of regular crews of this size.

3.2.7.3 Opportunity Cost of Additional Labour for Processes in which Operations may be performed in sequence or simultaneously

These processes include a wide range of farming activities. Characteristic equations take the form:

$$\left(\frac{A}{Y}\right)_1 + \text{--}\left(\frac{A}{Y}\right)_{S_1+1} + \text{--}\left(\frac{A}{Y}\right)_{S_1+S_2} = \left(\frac{A}{Y}\right)_{n-S_n+1}$$

$$+ \text{--}\left(\frac{A}{Y}\right)_n = T \qquad \text{.........} \ (3.41)$$

$$\sum_{i=1}^{N} S_i = n \qquad \text{.........} \ (3.42)$$

where:

n = number of operations to be performed in time T

N = number of workers

S_i = number of operations performed by one worker in time T

With the addition of a cost function to the qualifying equations (3.41, 3.42), a general solution should exist in Y_i, S_1 and N permitting the determination of a level value for the cost function.

There are strong reasons to believe that a solution can only be obtained by enumeration, i.e.: the number of sequences must be an integer; the number of operations in each sequence must also be an integer and the operations may be selected in any combination.

A feasible solution by enumeration can be obtained by:

1. Selecting the number of sequences.

2. Allotting operation to each sequence.

3. Solving for the optimum combination of machine sizes in each sequence by the method of Lagrange multipliers.

Lacking an algorithm by which succeeding iterations will always lead to an improved solution until a level value is reached, it is exceedingly important that every feasible solution be an attempt at the best solution.

An examination of the equations developed in leading up to the formulating of the general sequencing problem leads to a series of guiding principles that help to provide a rational basis for selecting the number of sequences (crew size) and for allotting operations to each sequence.

1. Operations should be divided into sequences that combine machines with high and low B' values rather than into sequences using machines with all low B' values and sequences using machines with all high B' values

2. Provided that tractor fixed-cost may be represented as: $C' = B'Y$
additional workers (and tractors) will not bring about a fixed-cost saving excepting that associated with the possible elimination of tractor overcapacity and that resulting from multiple shift operations of a tractor.

3. The lowest fixed-cost combination of machines is reached when each operation in a process takes time

T for its completion, i.e.: all operations are simultaneous rather than sequential.

4. The maximum* potential percentage saving in equipment fixed-costs by the addition of one worker equals

$$\frac{1}{N-1} - \frac{1}{N} \times 100\%$$

where N equals the new number of workers. e.g.: the maximum potential saving is 50% for a second man in the work force, a further 16% for a third, etc., until the conditions of (3) are reached, when no further saving is possible.

5. The saving in fixed-cost must exceed the cost of additional labour if there is to be economic justification for replacing sequenced operations by simultaneous ones.

6. The break-even point at present occurs when equipment with a replacement value of £4000 - 5000 is replaced by each full-time man.

An examination of (4), (5) and (6), particularly the geometric series in (6) reveals that, except for very large farms, workers in excess of one or two cannot be justified on the basis of the possible reduction in equipment fixed-cost through rearrangement of work schedules alone. A solution to Eq. (3.41) and (3.42) becomes largely an academic exercise.

3.2.7.4 Programming Crew-Size Problems

For farms where multiple-man crews are not essential to key processes, programs may be prepared as developed in Section 3.2.6.4. Where the resulting equipment combination cost (exclusive of tractors) exceeds £8000 - 10,000 (today's basis), the program should be re-run with new "t" values as determined in Section 3.2.7.3. (Western Canadian experience in this circumstance is reflected by the exceptionally high wages being paid for seasonal labour rather than by any increase in the regular labour force.)

Where multiple-man crews are thought to be essential to key processes, the following computational sequence may be followed:

* This maximum can only occur when all machines have equal B' values.

1. Program the farm as in Section 3.2.6.2 leaving out the enterprises requiring multiple-man crews.

2. Using the results of (1) as a first iteration, prepare new "t" values for the previous enterprise combination using the criteria of Section 3.2.7.3. (This is not a particularly difficult task since the crew-size has already been established.) Enter the processes requiring multiple-man crews and solve the program. Further iterations may be required to bracket the solution.

3. Compare the gross margin of (1) and (2).

3.3 CASE STUDIES

It was decided that case studies should be included in the investigation to provide the opportunity to work with real problems. The science of applied programming combines the development of mathematical expressions for real relationships in terms compatible with the mathematical model, and the simultaneous development of the model to permit the inclusion of the whole range of real variables. This implicit relationship between the problem and the model precludes the sequential development of theory and application. Many of the useful equations developed in Sections 3.1 and 3.2 arose out of problems presented by the case studies, rather than as preparation for the case studies. The presentation, however, appears more logical in its present sequencial form.

The farms selected for the case studies are representative of larger arable farms in the Lothians, but do not represent a statistical sample. All of the farms include provision for livestock feeding in their range of facilities. The programs are developed progressively. The basic variable - cost relationships for land use, livestock feeding, grazing and labour are developed and illustrated in the first two programs. Subsequent programs add fertility, structures, feed handling equipment, field machines and working force size to the basic formulation.

No "before" and "after" calculating of profitability is included, as such calculations would be irrelevant to the basic objective of presenting the problem and developing the model to permit a faithful representation.

3.3.1 CASE L-1

The farm specifications and equipment listing, together with general information reflecting the farmer's practices and preferences, were obtained from visits to the farm.

Land and buildings

Light arable	154 acres
Heavy arable	144 acres
Permanent pasture	45 acres
Total	343 acres
Cattle courts (open and closed)	4500 ft^2
Potato, turnip and grain storage	200 tons
Grain drying	None
Silos	Horizontal, unroofed flexible in size

Labour force

Farmer	1*
Working grieve	1
Tractor-men	2

* Farmer assists with farmstead operations, silage making and harvesting

Field machinery and equipment

Tractors	1 - Fordson Major 1 - Nuffield 2 - Ferg. 35
Plows	3 - 2-furrow
Drill	1 - 6" x 16
Roller, disc, harrows and sprayer	1 each
Combine	1 - Claas pull type
Forage harvester	1 - flail type

Baler	1 - low density
Trailers	2 - 3-ton
Potato planter	1 - 2-row - 3-man
Turnip planter	1 - precision
Potato digger	2 - 1 spinner 1 elevator
Buck rake and front end loader	1 each

Schedule of operations

This data has been combined with those from the other case study farms in Fig. 3.16.

Farming practices and preferences

Fertilizer was applied according to the general recommendations for the district. A 40-acre potato contract was held. All of the heavy land is suitable for growing potatoes, but only 100 acres of the light land. A normal distribution should, therefore, be 24 acres on heavy land, and 16 acres on light land. The farmer felt that no more than two-thirds of the arable acres should be sown to grain crops.

The farmer was willing to accept any beef enterprise that was profitable and also felt that an intensively grazed sheep enterprise should be considered. Pigs, dairying and poultry were ruled out by preference.

Assessment of the farmer

As with all of the farmers in this investigation, it was felt that a high degree of management skill had been demonstrated by past farming records and that this factor would not impose any limit on the realization of the potential of any enterprise combination.

Records of yield

The farmer's anticipation of crop yields were:

Wheat	L	36 cwt/acre
	H	40 cwt/acre
Barley	L	32 cwt/acre
	H	40 cwt/acre

Potatoes	L	10 T/acre
	H	12 T/acre
Roots	L	25 T/acre
	H	25 T/acre
Hay	L	1.75 T/acre
	H	2.5 T/acre
Silage	L	10 T/acre
	H	13 T/acre

Constraint No.	Requirement (Name)	Requirement (Number)	Inequality sign	Light Land Grass (A)	Silage (A)	Hay (A)	Wheat (A)	Barley (A)	Potatoes (A)	Roots (A)	Heavy Land Grass (A)	Silage (A)	Hay (A)	Wheat (A)	Barley (A)	Potatoes (A)	Roots (A)	Barley Sold (T)	Barley Bought (T)	Barley Cows (T)	Barley Calves (T)	Barley Fres (T)	Straw Cows (10T)	Straw Calves (10T)	Staw Fres (10T)	Silage Cows (10T)	Silage Calves (10T)	Silage Fres (10T)	Hay Cow (10T)	Hay Calves (10T)	Hay Fres (10T)	Roots Cows (10T)	Roots Calves (10T)	Roots Fres (10T)	Soyabeans Cows (Cwt)	Soyabeans Calves (Cwt)	Soyabeans Fres (Cwt)	Ewe	Cow	Calves	Fres (18 Mo)	Barley Beef	New Courts (£ x 100)	OVERTIME LABOUR March	April	Nov.
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	48
	Functional	0		6.9	8.1	5.1	-35.0	-1.25	-65	13.0	8.6	10.4	7.3	-38.8	-1.25	-75	13.0	-16.9	20.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.19	0.19	0.19	-9.65	-36.75	-29.25	-52.80	-45.8	7.9	2.9	2.9	2.9
1	Light Land Acres	154	≥	1	1	1	1	1	1	1																																				
2	" " Pots.	16	≥						1																																					
3	" " Bar.-Wh.	0	≥				1	-1																																						
4	" " Bar.+Wh.	103	≥				1	1																																						
5	Heavy Land Acres	144	≥								1	1	1	1	1	1	1																													
6	" " Pots.	24	≥													1																														
7	" " Bar.-Wh.	0	≥											1	-1																															
8	" " Bar.+Wh.	96	≥											1	1																															
9	Barley	0	≥					-1.6							-2.0			1	-1	1	1	1																				1.45				
10	Barley Straw	0	≥					-1.0							-1.2								1	1	1																					
11	Silage	0	≥	-.67	-10						-.85	-13														1	1	1										0.55								
12	Hay	0	≥	-.28		-1.75					-.35		-2.5																1	1	1										0.2					
13	Roots	0	≥							-25							-25															1	1	1												
14	Co SE	0	≥																	-15.7			-45.0			-24.6			-60.5			-15.7			-.715				169							
15	Co DCP	0	≥																	-15.7			-17.9			-40.3			-85.2			-13.5			-4.52				222							
16	Co DMFill	0	≥																	2.36			4.75			7.05			3.48			2.46			.117				-47.0							
17	Ca SE	0	≥																		-15.7			-45.0			-24.6			-60.5			-15.7			-.715				143						
18	Ca DCP	0	≥																		-15.7			-17.9			-40.3			-85.2			-13.5			-4.52				168						
19	Ca DMFill	0	≥																		2.36			4.75			7.05			3.48			2.46			.117				-31.6						
20	Fr SE	0	≥																			-157			-45.0			-24.6			-60.5			-15.7			-.715				220					
21	Fr DCP	0	≥																			-157			-17.9			-40.3			-85.2			-13.5			-4.52				212					
22	Fr DMFill	0	≥																			2.36			4.75			7.05			3.48			2.46			.117				46.7					
23	Grazing	15	≥	-.80		-.20					-1.0		-.25																									0.2	1.43		0.56					
24	Cattle Courts	450	≥																																				5	4	5	3	-10			
25	Capital	10000	≥																																			11.	70.	27.	33.	35.	55.			
26	Dec J F WD	470	≥					2.4	8.0	2.9					2.4	8.0	2.9																													
27	" " " Total	1410	≥					2.4	47.8	2.9						47.8																							5.4	5.4	8.1	2.7				
28	March WD	375	≥	0.6	0.6	0.6	0.8	2.2	7.1		0.6	0.6	0.6	0.8	2.2	7.1																							1.8	1.8	2.7	0.9		-10		
29	" OT	177	≥																																									+10		
30	April WD	375	≥						4.7	8.2						4.7	8.2																						0.9	0.9	2.7	0.9			-10	
31	" OT	177	≥																																										+10	
32	May WD	375	≥	0.5	8.8		0.2	0.2		6.0	0.6	11.0		0.2	0.2		6.0																									0.9				
33	" OT	177	≥																																											
34	June WD	375	≥	0.6		8.0			4.6	21.0	0.7		10.0			4.6	21.0																									0.9				
35	" OT	177	≥																																											
36	July Total	470	≥									6.0																														0.9				
37	Aug. WD	235	≥					1.2					5.0		1.2																											0.9				
38	" OT	235	≥																																											
39	Sept. WD	235	≥				3.5	2.6	4.0					3.5	2.6	4.0																														
40	" OT	235	≥																																											
41	Oct. WD	375	≥				3.9		14.0*	20.0**				3.9		16.0*	20.0**																													
42	" OT	177	≥																																											
43	Nov. WD	235	≥				0.7							0.7																																-10
44	" OT	120	≥																																											+10
45	" Total	470	≥				0.7							0.7																									0.9	1.8	2.7	0.9				

Staw Fres (10T)	Silage Cows (10T)	Silage Calves (10T)	Silage Fres (10T)	Hay Cow (10T)	Hay Calves (10T)	Hay Fres (10T)	Roots Cows (10T)	Roots Calves (10T)	Roots Fres (10T)	Soyabeans Cows (Cwt)	Soyabeans Calves (Cwt)	Soyabeans Fres (Cwt)	Ewe	Cow	Calves	Fres (18 Mo)	Barley Beef	New Courts (ft² x 100)	OVERTIME LABOUR March	April	Nov.	EXCHANGE LABOUR March	April	Nov.	CUSTOM LABOUR Dec. Jan. Feb.	March	Nov.
22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	48	49	50	56	57	58	65
0	0	0	0	0	0	0	0	0	0	0.19	0.19	0.19	-9.65	-36.75	-29.25	-52,80	-45.8	7.9	2.9	2.9	2.9	3.5	3.5	3.5	5.0	5.0	5.0
																	1.45										
1																											
	1	1	1										0.55														
				1	1	1										0.2											
							1	1	1																		
	-24.6			-60.5			-15.7			-.715				169													
	-40.3			-85.2			-13.5			-4.52				222													
	7.05			3.48			2.46			.117				-47.0													
		-24.6			-60.5			-15.7			-.715				143												
		-40.3			-85.2			-13.5			-4.52				168												
		7.05			3.48			2.46			.117				-31.6												
6.0			-24.6			-60.5			-15.7			-.715				220											
7.9			-40.3			-85.2			-13.5			-4.52				212											
.75			7.05			3.48			2.46			.117				46.7											
													0.2	1.43		0.56											
														5	4	5	3	-10									
													11.	70.	27.	33.	35.	55.									
																					-10					
														5.4	5.4	8.1	2.7								-10		
														1.8	1.8	2.7	0.9		-10			10	-10			-10	
																			+10								
														0.9	0.9	2.7	0.9			-10			10				
																				+10							
																	0.9										
																	0.9										
																	0.9										
																								-10			
																					-10			10			-10
																					+10						
															0.9	1.8	2.7	0.9									

An attempt was made to program the problem in such a manner as to permit the primary enterprises to be built up to the greatest possible extent from continuously variable components. Corolaries of the general proof for the simplex algorithm establish that where alternative enterprises are entered, with all common restraints, the number of those enterprises appearing in the solution will not exceed the number of restraints. This corolary was applied to preclude many pssible entries (and probably could have been applied to an even greater extent). The application of these principles in laying out a program generally conserves computer storage space. The introduction of digital computers with much faster "slow' storage permits very large matrices to be processed. These case study programs were, however, prepared for Ferranti Sirius and IBM 1620 computers. All of the programs as prepared run close to the limit of storage available in these computers, and it was therefore necessary to make the best use of available storage.

Rows 1 - 8, columns 1 - 14, provide the relationships limiting cropping and land use according to the physical boundaries of farm and the limitations imposed by the farmer. These entries are all straightforward. Rows 9 - 13 provide balance equations for barley, straw, silage, hay and roots. These equations are most useful, as replacement activities such as buying, selling and feeding can take place on a pure opportunity cost basis. The production of silage and hay from grazing land should be noticed. A full management system is implied by the grazing entry. In this case, one unit of grazing is made up three-quarters from three-year ley, and one-quarter from rye grass.

Rows 14, 15 and 16 provide for a least cost ration for winter feeding of suckler cows based upon the starch equivalent method of ration formulation. Data were obtained from Tables 3.1 and 3.2. The summer grazing requirement is based upon the system proposed by Holmes with acreage adjusted linearly with the energy requirement for cows and bullocks. Rows 17 - 22 provide a ration formulation for two other classes of livestock. It might be observed that a total ration for several classes of livestock could have been obtained by using only one set of three equations. The proportioning between classes would then provide a subsequent exercise. There is much to be said for this approach if the SE method is used, as feeds most suitable for maintenance and fat production could be allocated more appropriately. The "New Method" takes this factor into account, but as feeds show different energy levels for different levels of production, a total ration for lumped classes is not obtainable - nor would there be any reason for attempting to obtain it. The strength of the New Method depends upon its ability to most appropriately allocate available feeds.

Row 24 provides the requirement for court space. Column 40 permits the construction of new courts. It should be noticed that the cost row entry represents the yearly fixed-cost of investing in a unit area of court space, while $R_{25}C_{40}$ is the capital requirement for providing the unit of court space.

Row 25 provides a limit to investment in livestock and buildings. It has been shown[30] that a much more complete capital accounting can be provided for should this be felt necessary.

Rows 26 - 45 provide labour restrictions and requirements. Labour data were obtained from Appendix D. The relative values for total labour available, labour available for weather-dependent activities, and overtime are explained in Section 3.1. Columns 41 - 48 provide for overtime as a variable cost. Columns 49 - 56 permit operations to be delayed and completed the following month. The penalty for this practice was assessed as a labour charge per hour, midway between that incurred for overtime, and custom work. To insure that shortage of labour could not impose an artificial limit on the farming operation, an unlimited custom labour was permitted. The high charge for this labour, however, insures its proper place in farming practice.

Cost row entries were calculated as gross returns - variable costs. Many of the columns represent partial enterprises without marketable end products. The cost row entries for these activities simply represent the variable costs of the partial enterprise. Gross returns and variable costs were based upon current prices and indicated yields. Calculations are illustrated in Appendix E.

SOLUTION TO L-1

Gross Margin - £ 14,306

Rotations

Light Land	Grass	- 35 Acres
	Wheat	- 51.5 Acres
	Barley	- 51.5 Acres
	Potatoes	- 16 Acres
Heavy Land	Grass	- 14.5 Acres
	Silage	- 9.4 Acres
	Wheat	- 6 Acres
	Barley	- 90 Acres
	Potatoes	- 24 Acres

Livestock

Ewes 288	Pasture 5/Acre Feed 11 cwt. silage during winter
Barley Beef 180	29 cwt. barley plus regular feed supplements 925 ft sq of new courts required

Labour

Custom Labour equivalent to $1\frac{1}{2}$ regular workers required

Table 3.7

A summary of results for case study L-1 is shown in Table 3.7. No general significance is attached to these results, beyond showing that the program was possible; that a solution could be obtained using a standard library routine prepared for the Ferranti Sirius computer; and that the solution was reasonable. The clerical work associated with punching the program data on tape, together with checking it and correcting clerical errors, amounted to about two and one-half hours. The machine time required for the solution and read-out amounted to another hour and a half. Experience with this, and subsequent programs, indicated human failures not detected prior to the machine process in about one-third of the programs. These failures included counting errors (e.g., m x n in error), scaling errors (usually unscaled entries in a scaled row or column), ill conditioning, and in one instance, an infinite program.

Appendix H includes the complete computer print-out. The value of shadow prices in the interpretation of a program, as well as the value of other post-optimal information, was indicated in Section 2.

3.3.2 CASE L-2

Land and buildings

Good arable	525 acres
Trees, gardens, building	35 acres
Total	560 acres
Cattle courts	12,000 ft^2
Grain storage	Adequate
Grain drying	Yes
Potato storage	None
Silos	None

Labour force

Regular men (excluding farmer)	4
Potato harvesting and dressing	Casual

Field machinery and equipment

Tractors	1 - MF 65 4 - Fordson Dextra
Other machinery	To match

Straw handling by own baler

Large hand loaded trailer and hand stacking after harvest

Schedule of operations

Included in Fig. 3.16.

Farming practices and preferences

Fertilizers were applied according to the general recommendations for the district. A 70-acre potato contract was held. This was slightly exceeded some years, and not quite reached in others, depending upon the fields involved. An 8-year rotation had been followed, generally PC $\frac{C}{R}$ CC GGG. The farmer was willing to reduce the grass acreage to one year in eight if it could be shown to be profitable. 10 - 15 acres of turnips had been grown for sale. An additional acreage had been grown and folded with lambs. No rigid cattle policy had been established. The farmer preferred to keep the courts filled in winter, partly to provide dung for potato land. He operated a hill farm where a suckler herd and hill sheep flock were maintained. He preferred to think of this as a separate operation, and sold the hill calves whenever it was more profitable to purchase another class of store animal for the arable farm. Several classes of animals with which he was familiar and would consider feeding are shown in the completed matrix.

The farmer was not interested in dairying, poultry or pigs, but would consider increasing store lambs to fold on an additional acreage of turnips should this appear profitable. He had no silage making equipment, and was not convinced of any advantage of silage over hay.

Records of Yields

The farmer's anticipation of crop yields was:

Wheat	50 cwt/acre
Barley	39 cwt/acre
Roots	30 T/acre
Potatoes	13 T/acre
Hay	4 T/acre

Constraint No.		Requirement	Inequality Sign	GCC PC^C_RCCGGC GGG	C	R	G C C	G G C	G G G	Wheat (A)	Barley (A)	Roots Sold (10T)	Pasture (A)	Hay (A)	Barley Sold (T)	Barley Bought (T)	Hay Bought (T)	Hill Sheep (Score)
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	Functional	0		-9	0	1.3	.5	.8	1.1	-5.5	-.125	-3.0	0	.1	-1.69	2	1.2	-5
1	Arable acreage	525	≥	8														
2	3rd year balance	0	≥	-1	1	1												
3	6,7,8th year bal.	0	≥	-3			3	3	3									
4	Grain crops bal.	0	≥	-3	-1		-2	-1		1	1							
5	Roots (balance)	0	≥			-3.0						1.0						.92
6	Forage crops (bal.)	0	≥				-1	-2	-3				1	1				
7	Pasture balance	0	≥										-1					
8	Hay balance	0	≥											-4			-1	.6
9	Barley (balance)	0	≥							-1.95								
10	Stores & Hills	0	≥															
11	Hill Calves Heifs. SE	0	≥															
12	Hill Calves Heifs. DCP	0	≥												1	-1		
13	Hill Calves Heifs. DM	0	≥															
14	Hill Calves Steers SE	0	≥															
15	Hill Calves Steers DCP	0	≥															
16	Hill Calves Steers DM	0	≥															
17	Stores SE	0	≥															
18	Stores DCP	0	≥															
19	Stores DM	0	≥															
20	18 Mos. Fres. SE	0	≥															
21	18 Mos. Fres. DCP	0	≥															
22	18 Mos. Fres. DM	0	≥															
23	Roots Limit Heif.	0	≥															
24	Roots Limit Steers	0	≥															
25	Roots Limit Stores	0	≥															
26	Roots Limit Fres.	0	≥															
27	Bedding	0	≥							-1.75	-1.5							
28	Courts	12,000	≥															
29	Brock Potatoes	0	≥	-.15														
30	Limit Pot. Steers	0	≥															
31	Limit Pot. Stores	0	≥															
32	Limit Pot. Fres.	0	≥															
33	Capital balance	0	≥													2		
34	Dec.Jan.Feb. WD	78.5	≥	1.09		.29					.24							
35	Dec.Jan.Feb.Total	235.0	≥	3.98		.29					.24							
36	March WD	61.0	≥	.71						.08	.22		.06	.06				
37	March OT	30.5	≥															
38	April WD	61.0	≥	.47		.82							.05					
39	April OT	30.5	≥															
40	May WD	61.0	≥			.6				.02	.02							
41	May OT	30.5	≥															
42	June WD	61.0	≥	.46		2.1								1.05				
43	June OT	30.5	≥															
44	August WD	39.0	≥							.12								
45	August OT	39.0	≥															
46	September WD	39.0	≥	.6					.35	.26								
47	September OT	39.0	≥															
48	October WD	61.0	≥	1.8					.39									
49	October OT	30.5	≥															
50	November WD	50.0	≥						.07									
51	November OT	25.0	≥															

Constraint No.		Hill Heifers	Hill Steers	Stores (After Hill Steers)	Stores (Independent of Hill Cows)	Fresians (18 months)	Heifers: Hay (T)	Heifers: Barley (T)	Heifers: Roots (10T)	Heifers: Soya-meal (cwt)	Steers: Hay (T)	Steers: Barley (T)	Steers: Roots (10T)	Steers: Pota-toes (10T)	Steers: Soya-meal (cwt)
		16	17	18	19	20	21	22	23	24	25	26	27	28	29
	Functional	-3.2	-4.2	-3.2	-3.2	-5.7	0	0	0	.19	0	0	0	0	.19
1	Arable acreage														
2	3rd year balance														
3	6,7,8th year bal.														
4	Grain crops bal.														
5	Roots (balance)												1		
6	Forage crops (bal.)														
7	Pasture balance	.48	.52			.5									
8	Hay balance	-.25	-.25			-.5	1				1				
9	Barley (balance)							1							
10	Stores & Hills		-1	1											
11	Hill Calves Heifs. SE	10.1					-6.05	-15.7	-15.7	-.715					
12	Hill Calves Heifs. DCP	14.7					-8.5	-15.7	-13.5	-4.52		1			
13	Hill Calves Heifs. DM	-2.91					3.47	2.36	2.46	.117					
14	Hill Calves Steers SE		17.1								-6.05	-15.7	-15.7	-42.5	-.715
15	Hill Calves Steers DCP		20.7								-8.5	-15.7	-13.5	-24.7	-4.51
16	Hill Calves Steers DM		-9.11								3.47	2.36	2.46	5.9	.117
17	Stores SE			17.6	17.6										
18	Stores DCP			18.0	18.0										
19	Stores DM			-4.2	-4.2										
20	18 Mos. Fres. SE					22.									
21	18 Mos. Fres. DCP					21.2									
22	18 Mos. Fres. DM					-4.67									
23	Roots Limit Heif.	-1.5							10						
24	Roots Limit Steers		-2.4										10		
25	Roots Limit Stores			-2	-2										
26	Roots Limit Fres.					-2									
27	Bedding	1	1.5	1.25	1.25	1.5									
28	Courts	30	30+ 25	25	50	20+ 50									
29	Brock Potatoes													1	
30	Limit Pot. Steers		-.53											10	
31	Limit Pot. Stores			-1.6	-1.6										
32	Limit Pot. Fres.					-1.78									
33	Capital balance	4	4.3	5	5	4.5									
34	Dec.Jan.Feb. WD														
35	Dec.Jan.Feb.Total	.5	.667	.334	.5	.75									
36	March WD	.167		.167	.167	.21									
37	March OT														
38	April WD					.167									
39	April OT														
40	May WD														
41	May OT														
42	June WD	.125	.125			.25									
43	June OT														
44	August WD														
45	August OT														
46	September WD														
47	September OT														
48	October WD	.085	.167		.085	.5			2				2		
49	October OT														
50	November WD														
51	November OT														

Constraint No.		Stores: Hay (T)	Stores: Barley (T)	Stores: Roots (10T)	Stores: Pota-toes (10T)	Stores: Soya-meal (cwt)	Fresians: Hay (T)	Fresians: Barley (T)	Fresians: Roots (10T)	Fresians: Pota-toes (10T)	Fresians: Soya-meal (cwt)	Capital Req'd. (£ x 100)	Dec.,Jan.,Feb. (Cas.)	March OT	March (Cas.)	November OT	November (Cas.)
		30	31	32	33	34	35	36	37	38	39	40	41	42	43	56	57
	Functional	0	0	0	0	.19	0	0	0	0	.19	.07	.5	.375	.5	.375	.5
1	Arable acreage																
2	3rd year balance																
3	6,7,8th year bal.																
4	Grain crops bal.																
5	Roots (balance)				1				1								
6	Forage crops (bal.)																
7	Pasture balance																
8	Hay balance	1					1										
9	Barley (balance)																
10	Stores & Hills																
11	Hill Calves Heifs. SE																
12	Hill Calves Heifs. DCP		1					1									
13	Hill Calves Heifs. DM																
14	Hill Calves Steers SE																
15	Hill Calves Steers DCP																
16	Hill Calves Steers DM																
17	Stores SE	-6.05	-15.7	-15.7	-42.5	-.715											
18	Stores DCP	-8.5	-15.7	-13.5	-24.7	-4.52											
19	Stores DM	3.47	2.36	2.46	5.9	.117											
20	18 Mos. Fres. SE						-6.05	-15.7	-15.7	-42.5	-.715						
21	18 Mos. Fres. DCP						-8.5	-15.7	-13.5	-24.7	-4.52						
22	18 Mos. Fres. DM						3.47	2.36	2.46	5.9	.117						
23	Roots Limit Heif.																
24	Roots Limit Steers																
25	Roots Limit Stores			10													
26	Roots Limit Fres.								10								
27	Bedding																
28	Courts																
29	Brock Potatoes				1					1							
30	Limit Pot. Steers																
31	Limit Pot. Stores					10											
32	Limit Pot. Fres.									10							
33	Capital balance											-1					
34	Dec.Jan.Feb. WD												-1				
35	Dec.Jan.Feb.Total												-1				
36	March WD													-1	-1		
37	March OT													1			
38	April WD																
39	April OT																
40	May WD																
41	May OT																
42	June WD																
43	June OT																
44	August WD																
45	August OT																
46	September WD																
47	September OT																
48	October WD			2					2								
49	October OT																
50	November WD															-1	-1
51	November OT															1	

Rows 1 - 4, columns 1 - 8, provide the relationships from which a cropping pattern can be selected. The arrangement of restraints and requirements in this farm insures the retention of an 8-course rotation. The remainder of the program is prepared in much the same form as L-1. The farmer had definite ideas concerning the classes of livestock that he would consider for fattening. The inclusion of additional classes and sources of livestock, coupled with intake restrictions for potatoes and roots, result in a large matrix for the livestock portion. Limitation on matrix size, imposed by the Sirius computer, necessitated the omission of some of the labour portion from this program. Delaying operations was not permitted. However, the provision for unlimited custom labour prevented labour limitations from imposing an unrealistic barrier to development for the farm potential.

Assessment of the farmer

A high degree of management skill had been demonstrated by past farming records. It was felt that any combination of enterprises could be handled without difficulty.

SOLUTION TO L-2

Gross Margin - £32,985

Rotation: C C P C R C C G (65 Acres/break)

Wheat	328 Acres
Barley	6 Acres
Rotational grazing	45 Acres
Hay	20 Acres

Livestock:

Lambs - folded on turnips (65 acres) and hay	210 Score
Stores - (autumn purchased $6\frac{1}{2}$ cwt and finished in courts)	115

Rations for stores:

Barley	124 tons
Potatoes	13 tons
Protein conc.	18 cwt

Friesians - (bought at 200 lb, wintered in courts, grazed at 2/acre, then fattened in courts)	90

Rations for Friesians:

Barley	101 tons
Potatoes	85 tons
Protein conc.	21 cwt

Barley purchased:	225 tons
Capital for barley and livestock (excluding lambs)	£15,000

Table 3.8

A summary of results of the program prepared for case study L-2 is shown in Table 3.8. The complete computer read-out is included in Appendix H.

3.3.3 CASE L-3

Land and buildings

Heavy land		200 acres
Light arable land		200 acres
Raised beach		70 acres
	Total	470 acres
Cattle courts (open and covered)		12,700 ft^2
Convertible courts		7,200 ft^2
Potato storage, grain drying and storage		New building with ample capacity

Labour force

Tractor-men	6
Orramen	2
Stockmen	2*
Engineer	1
Women	2

* One stockman spends full time on a poultry enterprise that is not included in the linear program.

Field machinery and equipment

Tractors	4 - Fordson Major 2 - MF 35

Full line of machinery for root crops and field crops.

Sprinkler irrigation system to irrigate 200 acres of light series. (Water limits irrigable acreage to this level.)

Schedule of operations

Included in Fig. 3.16.

Farming practices and preferences

Early potatoes were grown in a 4-year rotation on the light land. Main crop potatoes constituted one year of a split six-year rotation on the heavy land. Three years of one-half of the six-year rotation was devoted to a 3-year gross ley. The farmer felt that the inclusion of the 3-year ley maintained the soil in sufficiently good condition to permit all of the straw to be removed from the cropped portion of this land to be tramped as dung and most of it applied to the light land to build up fertility and soil fibre.

The remainder of the dung, estimated as 12T per acre, was applied prior to the heavy land potato break in the cropping half of the split rotation. He felt that any cropping system that he adopted must maintain the fertility and texture in a manner equivalent to his present practice. Cattle numbers were closely related to the requirement for dung. A 50-cow suckler herd was outwintered on the raised beach. Calves, supplemented by purchased stores, were fattened in courts and were fed on byproducts (brock potatoes and beet tops) to as large an extent as possible. The farmer was interested in hearing the financial picture presented for other classes of livestock, including barley-beef, and the effect of the possible elimination of the suckler herd. He also wished to see the effects of permitting the 3-year ley to be replaced by another cropping practice, but one that would provide for an addition of nutrients and fibre to the soil equivalent to 12 tons of dung presently used in the rotation. Fertilizing for all crops was carried out at the level recommended for the area. The farmer wished to maintain a minimum of 60 acres of wheat to distribute the work load associated with grain cropping.

The farmer was not interested in dairy cows or pigs.

Estimate of crop yields

The farmer's estimate of crop yield, based upon farm records, was:

Early potatoes	7 T/acre
Main crop potatoes	12 T/acre
Wheat	40 cwt/acre
Barley	36 cwt/acre
Sugar beets	17 T/acre
Silage	15 T/acre

Constraint No.		Requirement	Inequality sign	CCGGGP	CCCCGP	P W Be W G W	2nd Year W	2nd Year G	3rd Year W	3rd Year Be	Silage (Acres)	Pasture (Acres)	Wheat (Acres)	Barley (Acres)	Wheat (Artificial)	Barley Sold (T)	Barley Bought (T)	Plowed Straw to 2	Plowed Straw to 3	Beet Tops (10T) Plowed In	Straw to Dung (T)	BB on Slats	BB in Courts	Fres. on Slats	Fres. in Courts	Sheep	Stores	Suckler Cows	Cows (Artificial)	Fresians Beet Tops (10T)	Fresians Pota-toes (10T)	Fresians Silage (10T)	Fresians Barley (T)	Fresians Soya-beans (cwt)	Stores Beet Tops (10T)	Stores Pota-toes (10T)	Stores Silage (10T)	Stores Barley (T)	Stores Soya-beans (cwt)	Sucklers Silage (10T)	Sucklers Barley (T)	Sucklers Soya-beans (cwt)	Commercial Fertilizers N (100 units)	Commercial Fertilizers P (100 units)	Commercial Fertilizers K (100 units)	New Courts Req. (100 ft.)	New Courts Slats (100 ft.)
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
	Functional			-7.4	-7.5	-7.6	0	0	0	-7	1.24	.86	-3.88	-.42	0	-1.85	2	0	0	0	.075	-4.57	-4.58	-5.75	-.58	-.965	-3.8	-7.32	0	0	0	0	0	.19	0	0	0	0	.19	0	0	.19	.27	.16	.165	.15	.9
1	Total Heavy Land	200	≥	6	6																																										
2	Total Light Land	200	≥			4																																									
3	2nd Year Balance	0	≥			-1	1	1																																							
4	3rd Year Balance	0	≥			-1			1	1																																					
5	18 Mos. Fres. SE	0	≥																					22	22					-20	-42.5	-24.6	-15.7	-.715													
6	18 Mos. Fres. DCP	0	≥																					21.2	21.2					-44.8	-24.7	-40.3	-15.7	-4.52													
7	18 Mos. Fres. DF	0	≥																					-4.67	-4.67					4.5	5.9	7.05	2.36	.117													
8	Stores SE	0	≥																								15.3								-20	-42.5	-24.6	-15.7	-.715								
9	Stores DCP	0	≥																								14.7								-44.8	-24.7	-40.3	-15.7	-4.52								
10	Stores DF	0	≥																								-4.2								4.5	5.9	7.05	2.36	.117								
11	Suck-lers SE	0	≥																									29.5												-24.6	-15.7	-.715					
12	Suck-lers DCP	0	≥																									37.2												-40.3	-15.7	-4.52					
13	Suck-lers DF	0	≥																									-7.9												7.05	2.36	.117					
14	NPK Bal-ance N	0	≥		8.7	8.7												4.00	4.00	-5.56	-.45	-2.68	-2.68							-5.56	-3.62	-4.65	-1.85	-.382	-5.56	-3.62	-4.65	-1.85	-.382	-4.65	-1.85	-.382	-10				
15	NPK Bal-ance P	0	≥		5.1	5.1												-.3	-.3	-1.87	-.3	-1.82	-1.82							-1.87	-2.68	-1.87	-1.26	-.109	-1.87	-2.68	-1.87	-1.26	-.109	-1.87	-1.26	-.109		-10			
16	NPK Bal-ance K	0	≥		20	20												-2.24	-2.24	-9.3	-2.24	-1.23	-1.79							-9.3	-13.4	-9.3	-1.23	-.246	-9.3	-13.4	-9.3	-1.23	-.246	-9.3	-1.23	-.246			-10		
17	NPK Bal-ance DM	0	≥		3.9	3.9												-.7	-.7	-.85	-.5	-.615	-.615							-.85	-1.2	-1	-.5	-.025	-.85	-1.2	-1	-.5	-.025	-1	-.5	-.025					
18	Straw Balance	0	≥		-5	-1.25	-1.25		-1.25									1	1		1																										
19	Straw to Dung	0	≥																		-1		1.5		1.5		1.5	1																			
20	Pasture & Silage	0	≥	-3	-1			-1			1	1																																			
21	Pasture Balance	0	≥									-1												.56	.56	.25		1.25																			
22	Silage Balance	0	≥								-1	-.1														.055						1.5					1.5			1.5							
23	Barley Balance	0	≥											-1.8		1	-1					1.45	1.45										1					1			1						
24	Straw to 2	0	≥		-5													1																													
25	Straw to 3	0	≥			-1.25	-1.25		-1.25										1																												
26	Wheat + Barley	0	≥	-2	-4	-1	-1		-1				1	1																																	
27	Wheat (minimum)	60	=										1		-1																																
28	Beet Tops (bal.)	0	≥					-12									10													10					10												
29	Beet Tops (max.)	0	≥																											10																	
30	Beet Tops (max.)	0	≥																					-1	-1										10												
31	Brock Potatoes (bal.)	0	≥	-.15	-.15	-.15																									1					1											
32	Courts	127	≥																				.75		.8		.8	.7																		-1	
33	Slats (balance)	0	≥																			.2		.25																							-1
34	New Courts & Slats	73	≥																																											1	1
35	Capital	15,000	≥																			35	35	45	45	11	50																			10	62.5
36	Cows (minimum)	50	=																									1	-1																		

The limitations imposed by the farm acreage and the rotating limits established by the farmer are provided by Rows 1 - 4, Columns 1 - 11.

Least cost ration formulations for three classes of livestock are provided by rows 5 - 13. Limits on the consumption of brock potatoes and beet tops are provided in rows 29 - 31.

Requirements for NPK and fibre to equal a 12-ton farm application are entered for rotations 2 and 3 in rows 14 - 17. The residual values of feeding stuffs for all components that might enter into rations and straw and beet tops are listed in appropriate columns in these tows. The method of calculation was developed in Section 3.1. As it was not considered practical to permit the transfer of straw from one field to another (except through dung), limits are imposed by rows 24 and 25. Columns 15 and 16 permit straw to be returned directly to the soil to help provide the fibre requirement. The additional nitrogen requirement to assist decomposition is charged against the straw. It may be made up from any nitrogen source, including an application of dung. A series of balance equations maintain factor relationships. The additional requirement for dung brought about by rotation 2 as an alternate to rotation 1 could not be met by existing court space under some permutations. In turn, straw for bedding might constitute a limit under some possible permutations even though court space was adequate. These possibilities were countered by provision to slat the floor of an available building, thus simultaneously removing both restrictions. As in case L-1, the yearly cost of the slats was assessed against the appropriate enterprises, and the entire capital requirement assessed against the available capital.

The considerable amount of space required for the development of the land use, feeding and fertility aspects of this farm precluded the possibility of including labour considerations. However, the complement of regular farm workers was such that the inclusion of labour would most likely have not affected the solution.

Assessment of the farmer

A high degree of management skill was apparent. It was felt that any combination of enterprises could be handled without difficulty.

SOLUTION TO L-3

(minimum 50 suckler cows)

Gross Margin - £ 25,028.40

Heavy Land rotation

C C C G G P (33.3 Acres/Field)

Light Land rotation

P C — 19 Acres / G — 31 Acres, Be C (50 Acres/Field)

Cropping:

Potatoes	83 Acres
Beets	50 Acres
Wheat	60 Acres
Barley	142 Acres
Grazing	64 Acres

Livestock:

Cows	50
Home reared store calves	45
Purchased stores	37
Barley beef in courts	95
Barley beef on slats	200

Rations for cows, calves and purchased stores:

Potatoes	125 Tons
Silage (pasture clippings)	42 Tons
Barley	*83 Tons
Protein concentrate	133 cwt
Cows and calves graze	64 Acres

All straw used for bedding.
Beet tops are ploughed in.
265 tons of barley are purchased.
1150 sq. ft. of new courts are required.
4000 sq. ft. of new slatted courts are required.

* No provision was made for purchasing hay in the program. A more acceptable ration could be obtained at no additional cost by purchasing some hay, rather than all barley.

Table 3.9

ALTERNATIVE SOLUTION TO L-3

(no minimum cow requirement)

Gross Margin - £ 25,822.60

Heavy Land rotation

C C C C G P (33.3 Acres/Field)

Light Land rotation

P C Be C (50 Acres/Field)

Cropping:	Potatoes	83 Acres
	Beets	50 Acres
	Wheat	60 Acres
	Barley	173 Acres
	Grazing	33 Acres

Livestock:

Cows	7
Home reared store calves	6
Purchased stores	25
Friesians (18 months)	44
Barley beef in courts	120
Barley beef on slats	153

Rations for cows, calves, purchased stores and Friesians (18 mths.)

Potatoes	124 Tons
Beet tops	69 Tons
Barley	*53 Tons
Silage (pasture clippings)	22 Tons
Protein concentrate	5 cwt

All straw used as bedding in courts.
Beet tops in excess of feeding requirements are ploughed in.
138 tons of barley are purchased.
2400 sq. ft. of new courts are required.
3000 sq. ft. of slatted courts are required.

* No provision was made for purchasing hay in the program. A more acceptable ration could be obtained at no extra cost by purchasing some hay, rather than all barley.

Table 3.10

As with case studies L-1 and L-2, this program was solved on a Ferranti Sirius computer, and required about two hours of computer time. The ease with which fixed-cost entries, exhibiting the characteristic equations of "buildings", can be combined with the variable-cost section of a farm business is demonstrated in this program.

The stringent requirements for "dung equivalent" was met only by increasing the court area. The relationship between areas of bedded and slatted courts also illustrates the value of combining the normally-fixed-cost section with the variable-cost section of a farm business.

Of interest from an agricultural science point of view is the considerable value of sludge in meeting the fibre requirement. The return of crop residue applied with an application of sludge should constitute an attractive alternative to tramped straw on many farms. The farmer's interest in comparing results with and without a minimum limit on the suckler herd required that two solutions be obtained. The step-function characteristic of this function is precisely the same as for a mechanized vs. hand operation for a "materials handling" problem. The method of solution, i.e.: program runs with and without the step function included, is, of course, also precisely the same as developed for materials handling problems.

3.3.4 CASE L-4

Land and Buildings

Arable land (med. through to heavy)	530 acres
Permanent pasture and buildings	20 acres
Total	550 acres
Cattle courts Bedded covered courts	16,800 ft^2
Grain and potato storage and grain drying	Adequate
Silo	Temporary horizontal silos used

Labour force

Grieve (working)	1
Tractor-men	6
Tractor-Cattleman	1
Orramen	2
Poultryman	1*
Gardener	1*
Total	12*

* Poultryman and gardener are not involved with enterprises included in the linear program.

Machinery and equipment

Tractors	2 - MF 65 *7 - MF 35
Grain combines	2 - MF 780
Unloading wagon	Gehl

* 3 used as spares

A line of machinery is available that permits all of the farm work with the exception of turnip hoeing to be carried out by the regular farm labour force.

Sechedule of Operations

Included in the preparation of Fig. 3.16.

Farming practices and preferences

The farmer held an 85 - 90-acre potato contract, and a 35-acre sugar beet contract. He had been including 2 years of grass in a 6-course rotation, but was willing to consider a reduction to one year if it could be justified economically.

Extensive new cattle courts had been built, and any combination of cattle enterprises that could be shown to be most economical was agreeable. He did not wish to extend the cattle courts further, however. Having provisions for drive-past bunks and an unloading wagon, the farmer felt that zero grazing might be practiced with grazing controlled by cutting some for silage. He wished to feed beet tops, if grown, for a period in early winter. The farmer was using an area of the new courts 20' x 140' for a silo. He was interested in the economics of building a new tower silo to permit the space occupied by the silage to be available for more livestock.

As the farm was large, requiring a large labour force, the farmer was anxious that a uniform work load be maintained as an aid to production control and the establishment of systematic routing for the performance of the farm work. The farmer's estimate of crop yields, based upon farm records, was:

Potatoes	12 T/acre
Sugar beets	13 T/acre
Turnips	25 T/acre
Wheat	2 T/acre
Barley	2 T/acre
Silage	13 T/acre

Assessment of the Farmer

A high degree of management skill was apparent. It was felt that any combination of enterprises could be handled without difficulty.

Constraint No.		Requirement	Inequality sign															Hill Calves - Summer					Hill Calves - Winter				18 Mos. Fres. Summer				18 Mos. Fres. Winter				Store Calves												
				CCPCCG	CCPGGG	B CCPCC R S	Beets (A)	Roots (A)	Silage (from 3) (A)	Barley (A)	Wheat (A)	Barley Sold (T)	Barley Bought (T)	Silage (A)	Mechanical Grazing (A)	Horizontal Silo (T)	Tower Silo (T)	Barley (T)	Straw Bought (T)	Silage (10T)	Grazing (A)	Soya-meal (cwt)	Barley (T)	Roots (10T)	Silage (10T)	Soya-meal (cwt)	Barley (T)	Silage (10T)	Grazing (A)	Soya-meal (cwt)	Barley (T)	Roots (10T)	Silage (10T)	Soya-meal (cwt)	Barley (T)	Roots (10T)	Silage (10T)	Soya-meal (cwt)	Hill Calves	18 Mos. Fres.	Store Calves	Barley-Beef	Dec.,Jan.,Feb. Labour	Mar.,Apr.,May Labour	June,July,Aug. Labour	Sept.,Oct.,Nov. Labour	
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	4
	Functional	0		-7.51	-7.52	-7.53	-5.5	1.3	0	-.125	-3.88	-1.69	2.0	1.04	0	.02	.064	0	.35	0	0	.19	0	0	0	.19	0	0	0	.19	0	0	0	.19	0	0	0	.19	-3.5	-5.28	-2.93	-4.58	.37	.37	.37	.55	
1	Crop Acres	530	≥	6	6	6																																									
2	Barley & Wheat	0	≥	-4	-2	-4				1	1																																				
3	Barley - Wheat	0	≥							-1	1																																				
4	Beets Roots Sz.	0	≥			-1	1	1	1																																						
5	Beets	35	≥				1																																								
6	Silage & Grazing	0	≥	-1	-3				-1					1	1																																
7	Silage Storage	0	≥											-1	-.33	.078	.068																														
8	Silage (balance)	0	≥													-1	-1			10					10			10					10				10										
9	Grazing (A)	20	≥												-1						1								1																		
10	Roots (T)	0	≥					-2.5																1.0								1.0				1.0											
11	Barley (T)	0	≥							-2		1	-1					1					1				1				1				1							1.45					
12	Hill Calves Sum. SE	0	≥															-15.7		-24.6	-20	-.715																			10.9								
13	DCP	0	≥															-15.7		-40.3	-50	-4.52																			10.9								
14	DF	0	≥															2.36		7.05	0	.117																			-3.26								
15	Wint. SE	0	≥																				-15.7	-15.7	-24.6	-.715															12.0								
16	DCP	0	≥																				-15.7	-13.5	-40.3	-4.52															16.4								
17	DF	0	≥																				2.36	2.46	7.05	.117															-3.24								
18	18 Mos. Fres. Sum. SE	0	≥																								-15.7	-24.6	-20.	-.715											9.8							
19	DCP	0	≥																								-15.7	-40.3	-50*	-4.52											13.1							
20	DF	0	≥																								2.36	7.05	0	.117											-2.26							
21	Wint. SE	0	≥																												-15.7	-15.7	-24.6	-.715							22.0							
22	DCP	0	≥																												-15.7	-13.5	-40.3	-4.52							21.2							
23	DF	0	≥																												2.36	2.46	7.05	.117							-4.67							
24	Store Calves SE	0	≥																																-15.7	-15.7	-24.6	-.715			13.3						
25	DCP	0	≥																																-15.7	-13.5	-40.3	-4.52			15.8						
26	DF	0	≥																																2.36	2.46	7.05	.117			-2.94						
27	Courts	16800	≥																																				.27	.5	.4	.35					
28	Bedding Straw	0	≥							-1.25	-1.25								-1																				.75	1	.8	.8					
29	Dec.Jan.Feb. Lab.	0	≥	3.98	3.98	3.98	1.71	.29		.24																													.15	.30	.198	.198	-1				
30	Mar.Apr.May Lab.	0	≥	1.58	1.58	1.58	4.18	1.9		.32	.133			1.55	.15					.015								.015													.08	.15	.10	.198		-1			
31	Jun.Jul.Aug. Lab.	0	≥	.62	.62	.62	3.64	2.8		.16					.3					.03								.03																.198			-1		
32	Sep.Oct.Nov. Lab.	0	≥	5.0	5.0	5.0	2.28	6.0		.26	.81				.15					-.15								.015													.08	.15	.10	.198				-1	
33	Total Labor	0	=	11.18	11.18	11.18	11.81	10.99		.98	.943			1.55	.6					.06								.06													.31	.60	.398	.792					-1
34	Dec.Jan.Feb. Def.	0	≥																																								1				-
35	Mar.Apr.May Def.	0	≥																																									1			-
36	Jun.Jul.Aug. Def.	0	≥																																										1		-
37	Sep.Oct.Nov. Def.	0	≥																																											1	-

* Estimated

tore Calves																
Roots (10T)	Silage (10T)	Soya-meal (cwt)	Hill Calves	18 Mos. Fres.	Store Calves	Barley-Beef	Dec., Jan., Feb. Labour	Mar., Apr., May Labour	June, July, Aug. Labour	Sept., Oct., Nov. Labour	Total Labour	(Artificial)	Dec., Jan., Feb. Casual Labour	Mar., Apr., May Casual Labour	June, July, Aug. Casual Labour	Sept., Oct., Nov. Casual Labour
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
0	0	.19	-3.5	-5.28	-2.93	-4.58	.37	.37	.37	.55	0	0	.37	.37	.37	.55
	10															
1.0																
						1.45										
			10.9													
			10.9													
			-3.26													
			12.0													
			16.4													
			-3.24													
				9.8												
				13.1												
				-2.26												
				22.0												
				21.2												
				-4.67												
-15.7	-24.6	-.715			13.3											
-13.5	-40.3	-4.52			15.8											
2.46	7.05	.117			-2.94											
			.27	.5	.4	.35										
			.75	1	.8	.8										
			.15	.30	.198	.198	-1									
			.08	.15	.10	.198		-1								
						.198			-1							
			.08	.15	.10	.198				-1						
			.31	.60	.398	.792					-1	-1				
							1				-.29		-1			
								1			-.29			-1		
									1		-.13				-1	
										1	-.29					-1

The matrix entries required to meet the limitations of acreage and preferences are straightforward. Similarly, the handling of ration formulation for 4 classes of livestock in addition to barley beef is the same as that developed for preceding programs. An examination of the capital cost of sealed silos and associated handling equipment follows:

Silo	11,000 cubic feet (245T)	£ 1985
	32,000 cubic feet (720T)	£ 3700
Unloader and Motors		£ 575
Blower		£ 275
2nd Storage Box		£ 575

It may be observed that the silo has a capacity vs. cost equation of

$$C_1 = £(1000 + 4T)$$

The addition of the auxiliary equipment results in an increment of

$$C_2 = £(1425)$$

If a useful life of 15 years can be expected for the silo, and 7 years for the mechanical equipment, an expression for yearly cost becomes

$$C' = £(304 + 0.27T)$$

Results of tests on silage losses[40] indicate a loss of feeding value in sealed silos of about 10%, and in horizontal silos 30%. Using these figures, the change from a bunker silo to a sealed tower silo would be equivalent to an increase in silage available for feeding from 13T/acre to 16.7T/acre, i.e.,

$$\frac{90}{70} \times 13 = 16.7\text{T/acre}$$

The operation would not be materially affected by the change to an upright silo. Loading of the trailer would become a simpler operation, but not one that would save a great deal of time; silo filling time would not be appreciably altered. The significant entries for the linear program, should the project be considered, are then

1. Release of 2,800 ft^2 of court space.
2. Increase in yearly fixed-cost of £(304 + 0.27T).
3. An "effective" increase in silage yield from 13 tons to 16.7 tons per acre.

There are two ways of entering this problem. Two solutions can be obtained, or the yearly fixed-cost curve can be approximated as a linear function of quantity of silage. This involves estimating the silage production and calculating a linear function that will agree with the real function in the region of the solution. This method, illustrated in Fig. 3.22, was used in solving this program.

SOLUTION TO L-4

Gross Margin* -	£ 24,094.1	
Rotation	C C C C G P (88 Acres/break)	
Cropping	Wheat	24 acres
	Barley	279 acres
	Rotational Grazing	75 acres
	Permanent Grazing	26 acres
	Silage	13 acres (plus pasture clippings)
Livestock	Barley Beef	345
	Spring Calves - purchased Sept.-Nov., wintered in courts fattened on grass	175
	Feed for wintering calves:	
	Barley	57 tons
	Silage	488 tons
Silo	Build bunker silo	

* Gross returns less variable costs (purchased feeds, seeds, fertilizers, etc.) and also less all labour costs.

Table 3.11

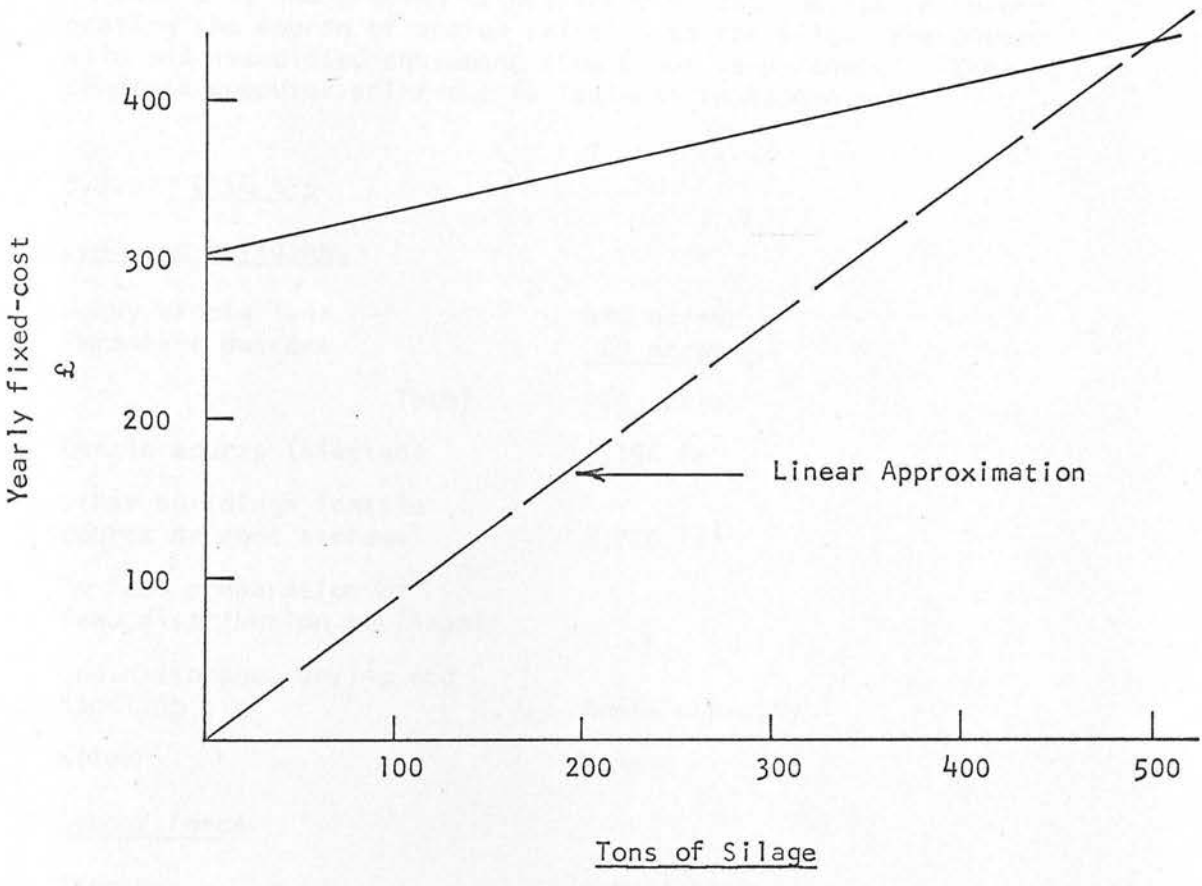
Yearly fixed-cost
£
400
300
200
100
100
200
300
400
500
Linear Approximation
Tons of Silage

Fig. 3.22

The completed matrix includes all of these aspects of the proposed project.

Labour requirements and restrictions are entered to exert an influence in the selection of enterprises so as to level out the seasonal work load. Labour imbalance results in the hiring of "costly" custom labour.

Table 3.11 lists the activities that constitute the solution to Case L-4. No problem is presented by the results in interpreting the course of action relating to the silo. The tower silo and associated equipment should not be purchased. The complete computer print-out is included in Appendix H.

3.3.5 CASE L-5

Land and buildings

Heavy arable land	400 acres
Permanent pasture	29 acres
Total	429 acres
Cattle courts (slatted)	1,790 ft^2
Other buildings (cattle courts or root storage)	5,700 ft^2
No feed preparation or feed distribution equipment	
Grain storage, drying and handling	Ample capacity
Silos	None

Labour force

Stockmen	1 full time
Tractor-men and others	5 or 6
Total	6 or 7

Field machinery and equipment

Full line of machinery for cereal cropping and row cropping.

No silage making equipment.

Schedule of operations

Included in the tabulations for Fig. 3.16.

Farming practices and preferences

The farmer practiced a seven-course rotation including a potato course. A 57-acre potato quota was held. The grain-potato rotation was broken with a rye grass course, and a split kale-root course. The farmer felt that the grass was necessary in his rotation, but did not wish to invest in silage making and handling equipment because he was not convinced that it was economic to do so. He therefore leased the grass as grazing to a sheep farmer. The kale and roots were folded as part of the grazing contract. The farm buildings were extensive, but of awkward shapes for remodelling to enable modern livestock handling practices. The farmer considered that best use could be made of the existing buildings by slatting the floors and restricting the livestock enterprise to barley beef. A full-time herdsman was employed, and the size of the barley beef enterprise was restricted to the number of animals that the cattleman could manage. No mechanization was provided for either feed mixing or feed distribution. Acceptable alternatives were discussed with the farmer. He was willing to consider an additional cattle enterprise that would permit the utilization of the grass acreage, provided that it would fit in with his belief in slats, and could be managed by his present cattleman. He felt that only a completely mechanized operation could be considered as acceptable if silage was involved. The farmer was also prepared to increase the size of his barley beef enterprise by installing a feed mixer, or an auger to fill the feeders, or both, if these additions were economically sound. He felt that in any case the cattleman should not be actively working for more than 2/3 of the working day, to permit close observation of the cattle and attention to odd jobs associated with the barley beef enterprise. The farmer felt that the risks of loss associated with cattle feeding of all types should limit his investment in livestock to £10,000.

Estimated yields

The farmer's estimate of crop yields, based upon farm records, was:

Main crop potatoes	12 T/acre
Wheat	2 T/acre
Barley	2 T/acre
Silage	12 T/acre
	(15.5 with sealed silo)

Assessment of the farmer

It was felt that any combination of enterprises falling within the general pattern of possibilities which were discussed could be managed at full potential.

Constraint No.		Requirement	Inequality sign	GPCC^K_RCC	GCCCPCC	PGGGCCC	Wheat (A)	Barley (A)	Grazing (A)	Silage (A)	Barley Sold (T)	Barley Bought (T)	Barley to Fres. (T)	Silage to Fres. (10T)	Soyabeans to Fres. (cwt)	Barley to Calves (T)	Silage to Calves (10T)	Soyabeans to Calves (cwt)	Sheep - Grass Kale	Fresians	Calves	1 Barley Beef	2 Barley Beef	3 Barley Beef	New Courts	Grazing Artificial
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
	Functional			* -53.7	* -51.4	* -49.4	-38.8	-1.25	0	1.8	-16.9	20.0	0	2.7	1.90	0	2.7	1.90	-2.1	-50.7	-31.9	-45.4	-45.26	-45.09	7.5	0
1	Crop Acres	400	≥	7	7	7																				
2	Barley & Wheat	0	≥	-4	-5	-3	1	1																		
3	Barley - Wheat	0	≥				1	-1																		
4	Grazing & Silage	29	≥		-1	-3			1	1																
5	Grazing	29	=						1																	-1
6	Silage (T)	0	≥						-.7	-15.5				10			10									
7	Grazing	0	≥						-1											.56	.75					
8	Barley (T)	0	≥					-2			1	-1	1			1						1.45	1.45	145		
9	18 Mos. Fres. SE	0	≥										-15.7	-24.6	-.715					22						
10	18 Mos. Fres. DCP	0	≥										-15.7	-40.3	-4.52					21.2						
11	18 Mos. Fres. DM	0	≥										2.36	7.05	.117					-4.67						
12	Calves SE	0	≥													-15.7	-24.6	-.715			9.0					
13	Calves DCP	0	≥													-15.7	-4.52	-4.52			12.3					
14	Calves DM	0	≥													2.36	.117	.117			-2.43					
15	Courts	1,790	≥																	25	20	20	20	20	100	
16	New Courts	7,200	≥																						100	
17	1 limit	120	≥																			1				
18	2 limit	120	≥																				1			
19	3 limit	160	≥																					1		
20		0	≥	-1																						
21	Min Hay	0	≥														-10				1					
22	Min Hay	0	≥											-10						1						
23	Dec. J F WD	590	≥	8	8	8		2.4																		
24	Dec. J F Total	1,888	≥	47.8	47.8	47.8		2.4																		
25	March WD	500	≥	7.1	7.1	7.1	.8	2.2	.6	.6																
26	March OT	250	≥																							
27	April WD	500	≥	4.7	4.7	4.7																				
28	April OT	250	≥																							
29	May WD	500	≥				.2	.2	.5																	
30	May OT	250	≥																							
31	June WD	500	≥	4.6	4.6	4.6			.6	8.0																
32	June OT	250	≥																							
33	July Total	590	≥																							
34	August WD	295	≥					1.2																		
35	August OT	295	≥																							
36	September WD	295	≥	4.0	4.0	4.0	3.5	2.6																		
37	September OT	295	≥																							
38	October WD	500	≥	14.0	14.0	14.0	3.9																			
39	October OT	250	≥																							
40	November WD	295	≥				.7																			
41	November OT	150	≥																							
42	November Total	590	≥				.7																			

Constraint No.		Overtime								Casual									
		March	April	May	June	August	September	October	November	Dec., Jan., Feb.	March	April	May	June	July	August	September	October	November
		24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41
	Functional	2.9	2.9	2.9	2.9	2.9	2.9	2.9	2.9	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
23	Dec. J F WD									-10									
24	Dec. J F Total									-10									
25	March WD	-10									-10								
26	March OT	10																	
27	April WD		-10									-10							
28	April OT		-10																
29	May WD			-10									-10						
30	May OT			10															
31	June WD				-10									10					
32	June OT				10														
33	July Total														-10				
34	August WD					-10										-10			
35	August OT					10													
36	September WD						-10										-10		
37	September OT						10												
38	October WD							-10										-10	
39	October OT							10											
40	November WD								-10										-10
41	November OT								10										
42	November Total								-10										-10

* Variable cost of roots and grass deducted.

The matrix entries required to meet the limitations imposed by acreage and rotational preferences are straightforward. Ration formulation is handled in the manner developed for the preceding case studies. It is necessary, however, that the rations include only silage, barley and protein supplement to permit completely mechanized handling and to prevent trouble with the slats.

Capital cost consideration involves silage making, storage and distribution auger for barley beef, and additional areas of slatted floor. An assessment of the problem indicated that, as with preceding case studies, the slatted floor could be handled as a linear function of enterprise size. Two possibilities exist for silage: either to have, or not to have, silage handling equipment. Three possibilities exist for barley beef equipment: no change; addition of an auger; and addition of a mixer. If each of the possibilities is added as a step function, there would be 6 possible combinations, and it would therefore be necessary to solve the program 6 times. It was decided, because of the relatively low capital outlay required, to express the options for barley beef as an incremental approximation to a curvilinear function.

The considerable outlay required to permit the handling of silage precluded this possibility, so two solutions were still required - one with silage included in the program, and one with silage not included.

Capital cost requirements for silage equipment were:

Silo, as in Case L-4

$$C = £(1000 + 4T)$$

Other silage handling and feeding equipment

(4)	Unloader and motor	£575
(3)	Blower	£275
(2)	Blower feeder	£475
(1)	Forage harvester with attachments	£1,015
	Auger conveyor	£50
	Feeder auger head	£70
	Distribution auger and bunk	£5/animal
	Total	£(2460 + 5/animal)

Based upon a 15-year estimated useful life for the silo, and a 7-year estimated useful life for the mechanized equipment, the yearly capital cost

$$C^1 = £(418 + .27 \text{ Ton} + .7 \text{ Animal})$$

Capital cost requirements for changes in the barley beef feeding equipment were:

Feed mixer (5 cwt)	£ 130
Distribution auger (150 ft)	£ 350

plus increment in cost of mixer.

Yearly capital cost for the mixer, based upon a 7-year useful life, is

$$C^1 = \frac{130}{7} = £\,18.6/\text{year}$$

Yearly capital cost for the auger system is

$$C^1 = \frac{350}{7} = £\,50/\text{year}$$

Observed times for the operations performed by the cattleman while preparing and distributing feed by hand were:

Hand mixing 4 - 5 cwt of barley and concentrate, bagging, and loading barrow	25 minutes
Wheeling barrow to courts, carrying bags to feeders, emptying and returning to preparation area	15 minutes

Observations of a 5 cwt mixer indicated that the time required in the mixing area would be reduced by 20 minutes per batch of 4 - 5 cwt. Observations of another installation indicated the addition of a distribution auger, coupled with an auger batch mixer, would reduce the handling time by another large increment. Translating this time study information to handling rate indicates that, without mechanization, the time available (4 hours per day) limits the enterprise size to 120 animals. The addition of the mixer doubles this figure to 240 animals per day. The addition of the auger would permit all of the available court space to be filled, i.e., approximately 340 animals could be housed. Related to the auger, this limit could be put on enterprise size.

These limits on enterprise size, coupled with the capital cost figures, permit enterprise size to be represented as an incremental summation.

		$\Delta_1 A$	$\Delta_2 A$	$\Delta_3 A$
120	$\geq$	1		
120	$\geq$		1	
100	$\geq$			1
Z	=	0	$-- \frac{18.6}{120}$	$-- \frac{50}{100}$

Fig. 3.23

This is illustrated in Fig. 3.23. This set of equations is included in the matrix for L-5; the cost-row entries, of course, combine the entries shown in Fig. 3.23 with the normal variable-cost calculations.

All other entries in the matrix for L-5 are straightforward. Purchased hay pellets were offered in place of silage in L-5-2 to retain the objectives of mechanized feeding without withdrawing roughage from the ration.

The solutions, one for the program without a silo, and the other for the program with a silo included, are shown in summary in Tables 3.12 and 3.13 respectively. The difference in gross margin (22,092.8 - 20,876.5) exceeds the constant term of the silo fixed-cost (418), so the recommendation is clearly to construct the silo and follow the program indicated in Table 3.13.

Both programs show that the barley beef enterprise should be mechanized to permit the size of the enterprise to be increased. In this case both capital improvements are indicated.

The computer print-out for both programs is shown in Appendix H.

SOLUTION TO L-5

WITHOUT SILO

Gross Margin - £20,876.54

Rotation: G C C C P C C (57 acres/break)

Potatoes	57 acres
Wheat	14 acres
Barley	271 acres
Grazing	
- rotational	57 acres
- permanent	29 acres

Livestock:

Calves (wintered, then finished on grass)	114
Feed - 11T hay pellets 57T barley 92 cwt soyameal	
Barley beef	334 per year

Additional Courts: 7,200 ft^2

Table 3.12

SOLUTION TO L-5

WITH SILO

Gross Margin - 22,092.80

Rotation: G C C P C C (57 acres/break)

	Potatoes	57 acres
	Wheat	16 acres
	Barley	269 acres
	Grazing - rotational	31 acres
	- permanent	29 acres
	Silage (pasture clippings in addition)	26 acres
Livestock:	Fresians (bought at 200 pounds, wintered, pastured, then finished during second winter)	108
	Feed - 439T Silage, 83T Barley	
	Barley beef	314 per year
Additional Courts:		7,200 ft^2

Table 3.13

The solutions, one for the program without a silo, and the other for the program with the silo, are shown in summary in Tables 3.12 and 3.13 respectively. The difference in gross margin (22,092.8 - 20,876.5) exceeds the constant term of the silo fixed-cost (418), so the recommendation is clearly to construct the silo and follow the program indicated in Table 3.13.

Both programs show that the barley beef enterprise should be mechanized to permit the size of the enterprise to be increased. In this case both capital improvements are indicated.

The computer print-out for both programs is shown in Appendix H.

3.3.6 CASE L-6

Land and buildings

Good arable land		260 acres
Sandy raised beach		83 acres
	Total	343 acres
Cattle courts, new building		4,400 ft^2

Other buildings - good pig buildings; horizontal silo; grain storage in a variety of buildings; no facilities for drying; good potato store

Labour force

Tractor-men		4
Others		1
	Total	5

Machinery and equipment

Tractors 2 - Nuffield
2 - Ferg. 35

Beet harvester

Potato harvester

Full line of complementary equipment

Schedule of operations

The schedule of operations was similar to that illustrated in Fig. 3.16, except that the early potatoes required irrigation throughout the month of June, and the early potato harvest was carried out in July. It was considered essential that the early potatoes be picked by hand, using squad labour.

Record of yields

The farmer's anticipation of crop yields was:

Potatoes	
Main crop	12 T/acre
Earlies	$7\frac{1}{2}$ T/acre
Wheat	2 T/acre
Barley	2 T/acre
Silage	15 T/acre
Beets	14 T/acre
Mangles	30 T/acre

Farming practices and preferences

The farmer practiced a seven-course rotation, G.G., W or B, Pot, W, Beets*, W. He grew earlies in approximately one-half of the 37-acre potato break, and main crop in the remainder. He was interested in increasing the proportion of earlies provided that the cost of a new main irrigation line could be offset by the increase in returns from potatoes. The farmer was also willing to reduce his grass to a one-year stand provided this acreage could be grazed or used for silage only. He was not interested in barley beef animals, but would consider any conventional beef feeding project. The area of blow sand was being regenerated by grazing bullocks. The farmer felt that control of numbers between 20 and 50 would provide most rapid rejuvenation. The pig enterprise was considered to be fixed, and was not to be included in the program. One man (of the 5) worked full time looking after the pigs, and a barley supply of 195 tons per year was required for pig feed.

Assessment of the farmer

It was felt that the farmer could manage any combination of enterprises permitted by the program.

A general assessment of the farming operation, however, suggested that the small arable acreage probably did not justify the size of crew (4 tractor-men) and that this in turn could only be reasonably changed significantly by changing the pattern of farming. It was further felt that the amount of machinery required to operate the diversified operation probably further mitigated against this type of operation on the

* Mangles are sown to a total of 4 - 5 acres with the beets.

limited scale possible. It was decided, therefore, to consider an alternative farming practice, and to include the effect of machinery fixed-cost in selecting an enterprise combination. The alternative to conventional (for the district) practice was considered to be the elimination of row crops and a reduction of the labour force to 1 man, assisted by the farmer during combining, and by the farmer and casual labour during silage making.

Program with a one-man working force

Based upon rotations that were acceptable to the farmer, it was guessed that the cropping program would be as follows:

Barley acres	=	108
Wheat acres	=	108
Silage acres	=	44
Store cattle and Friesians	=	88

From Fig. 3.16, and using adjusted weather data, the following qualifying equations can be written

March
Planting Barley and Feeding Stores

$$\frac{108}{Y_D} + \frac{108}{Y_S} + \frac{108}{Y_R} + \frac{1320^{*}}{Y_A} \leq 185 \tag{1}$$

August
Combining Barley

$$\frac{108}{Y_C} + \frac{108}{Y_B} \leq 156 \tag{2}$$

September
Combining Wheat

$$\frac{108}{Y_P} + \frac{108}{Y_B} \leq 156 \tag{3}$$

Oct.-Nov.
Planting Wheat and Feeding Stores

$$\frac{108}{Y_P} + \frac{108}{Y_D} + \frac{108}{Y_B} + \frac{108}{Y_R} + \frac{2200^{**}}{Y_A} \leq 302 \tag{4}$$

* 88 animals per day for 15 days.

** 88 animals per day for 25 days.

Dec.-Jan.* Plowing Barley Land	$\frac{108}{Y_P}$	$\leq$	156	(5)

It becomes obvious from examination that qualifying Eqs. (2) and (4) will be limiting.

There are no common variables so that the equations may be treated separately.

Considering Eq. (2): From Appendix F, the cost coefficient of combine size is £45 per ton per hour. Related to estimated yield for farm L-6 of 2 tons per acre, the cost coefficient may be expressed as £90 per acre per hour.

A two-man bale-handling system manufactured by Farmhand Inc. shows real promise in easing the task of handling bales, the system consisting of a bale accumulator to be pulled (without attendant) behind a baler, a grapple fork device to handle bales in bunches of 8, and a regular front end loader frame. Bales are picked up in units of 8 and placed on a flat bed in a manner similar to pallet handling. The bales are removed (in 8's) and stacked by the same fork cum front end loader. The Canadian cost of the system is \$2,344 (not including the tractor or trailer or baler). Capacity has been measured by the Agricultural Machinery Administration, Province of Saskatchewan, as 250 bales per hour, picked up, transported and stacked. From this data, it can be calculated that if this operation follows a field baling operation, (rather than taking place simultaneously) a combined capacity of about 1.5 acres per hour of straw is reasonable. Assuming a ten-year depreciation period, a cost coefficient can be calculated as

$$B' = \frac{\$2344 + \$1000^{**}}{\$3/£ \times 10 \text{ years}} \times 1\tfrac{1}{2} \text{ acres/hour}$$

$$= £\ 74 \text{ per acre per hour}$$

* It was considered that there would be no competition between feeding and plowing.

** portion of baler assessed to this operation.

It should be observed that this B' value occurs at a point, rather than over an interval. However, entering this number as a linear coefficient may be justified in that it permits this function to be considered in association with continuously variable functions. If a solution falls away from the point at which the value applies, then reconsideration is necessary.

With this qualification a functional may be written as:

$$C' = 90Y_C + 74Y_B \qquad (A)$$

Equation (2) constitutes a qualifying equation that may be written as:

$$\frac{1}{Y_C} + \frac{1}{Y_B} = 1.44 \qquad (B)$$

It was shown in Section 3.2 by the method of Lagrange's multiplier that a minimum cost combination is obtained when -

$$Y_j = \frac{\sum_{i=1}^{h} \sqrt{Y_i}}{\sqrt{Y_j}} \quad \frac{A}{T}$$

Solving this particular problem,

$$Y_c = \frac{\sqrt{90} + \sqrt{74}}{\sqrt{90}} \quad \frac{1}{1.44}$$

$$= 1.32 \text{ acres per hour}$$

$$Y_B = \frac{\sqrt{90} + \sqrt{74}}{\sqrt{74}} \quad \frac{1}{1.44}$$

$$= 1.46 \text{ acres per hour}$$

The hours spent on each activity can be calculated as

Combingin barley	82
Handling straw	74

These times are inserted in the appropriate spaces in program L-6.

Considering Eq. (4), a cost function may be written as:

$$C' = B'_P Y_P + B'_D Y_D + B'_B Y_B + B'_R Y_R + B'_A Y_A$$

Reference 25 lists the "acres per hour per foot of width" capacities of the field machines as 0.18, 0.25, 0.25 and 0.28 respectively. These figures may be combined with data from Appendix H and a 10-year useful life to yield the following B' values

$$B'_P = £20/\text{acre/hr/year}$$

$$B'_D = £8/\text{acre/hr/year}$$

$$B'_S = £8/\text{acre/hr/year}$$

$$B'_R = £3.6/\text{acre/hr/year}$$

If a tractor is added and an 8-year useful life assumed, the B' value is calculated as:

$$B'_T = £2.75/\text{hp/year}$$

The data from program L-5 yields a total yearly cost figure for a mechanized feeding system of:

$$C' = £(418 + 0.27T + 0.7N)$$

for 88 animals. This becomes

$$C' = 418 + (0.27 \times 5 \times 88) + (0.7 \times 88) = £598$$

Assuming that the silo unloader operates at a rate of 150 pounds of silage per minute, the effective capacity of a system using an unloader is about 100 animals per hour, per day. This figure can be combined with the C' value to establish one point on a capital cost or capacity curve.

B' = £5.98/year/animal per hour

Other points may be plotted for other feeding methods, e.g., hand feeding, and unloading wagons from horizontal silos. Even without these other points, the argument presented for the bale handling system may be advanced to justify the inclusion of this figure in a cost equation.

A complete cost equation may now be written as:

$$C' = 20Y_P + 8Y_D + 8Y_S + 3.6Y_R + 2.75Y_T + 5.98Y_F \qquad (C)$$

Draft figures for machinery obtained from reference

plow	900 lb/ft of width (6" deep)
disc (double)	150 lb/ft of width
drill	80 lb/ft of width
roller	60 lb/ft of width

Relating these figures to the normal operating speeds listed in reference 25 as

plow	$2\frac{1}{2}$ MPH
disc	3 MPH
drill	3 MPH
roller	$3\frac{1}{2}$ MPH

yields power requirements of

plow	6 HP/ft
disc	1.2 HP/ft
drill	0.6 HP/ft
roller	0.5 HP/ft

Assuming an average tractor loading of 50% of rated drawbar horsepower, the tractor requirements are:

plow	12 HP/ft
disc	2.4 HP/ft
drill	1.2 HP/ft
roller	1.0 HP/ft

Relating these requirements to capacities in acres-per-hour yield, the following qualifying equations

$$\left.\begin{aligned} Y_T &\geq 66\ Y_P \\ Y_T &\geq 9.6\ Y_D \\ Y_T &\geq 4.8\ Y_S \end{aligned}\right) \qquad \text{(D)}$$

$$Y_T \geq 3.6\ Y_R$$

Equations (4), (C) and (D) may be combined to yield a least-cost combination. In this particular case, the method of Lagrange's multipliers was used by considering (D) 1 as an equality;* adding slack variables to (D) 2, 3, and 4, and giving these slack variables a small cost coefficient in (C). The solution was simplified by changing the entry in (4) from $\frac{2200}{Y_F}$, to $\frac{108}{Y'_F}$, and making the necessary cost coefficient change in (C) from 5.98 to $5.98 \times \frac{2200}{108}$.

* The application of the method of Lagrange's multipliers seems to break down occasionally, and some slightly different arrangement of the qualifying equations is required. Theory on the method[16,44,49] mentions this possibility, but provides no guide to predictability.

The solution obtained was:

$$Y_T = 54.5 \text{ HP}$$

$$Y_P = 0.825 \text{ acres/hour}$$

$$Y_D = 4.10 \text{ acres/hour}$$

$$Y_S = 4.10 \text{ acres/hour}$$

$$Y_R = 6.15 \text{ acres/hour}$$

$$Y_F = 21.6 \text{ animals/hour}$$

from which the hours spent at each activity are calculated as:

Plowing	131.0
Discing	26.4
Seeding	26.4
Rolling	17.6
Feeding Cattle	102.0 hours in 25 days, or
	4.1 hours per day

These times are inserted in the appropriate spaces in program L-6 One-Man Working Force.

This program was solved on an IBM 1620 computer using a standard library program for linear programming. The solution is summarized in Table 3.14. The computer read-out is contained in Appendix H.

ONE-MAN WORKING FORCE

Constraint No.	Requirement (Name)	Requirement (Number)	Inequality sign	G C C C C C	G G G C C C	Wheat (Acres)	Barley (Acres)	Straw Handled (T)	Straw Sold (T)	Barley Bought (T)	Barley Sold (T)	Barley to Pigs (T)	Stores on Sand	Stores (Artificial)	Stores in Courts	18 Mos. Fresians	N	P	K	DM (Artificial)	Stores on Sand: Silage (Acre = 15T)	Stores on Sand: Straw (T)	Stores on Sand: Soya-meal (cwt)	Stores in Courts: Silage (Acre = 15T)	Stores in Courts: Hay Purchased (T)	Stores in Courts: Barley (T)	Stores in Courts: Soya-meal (cwt)	18 Mos. Fres.: Silage (Acre = 15T)	18 Mos. Fres.: Barley (T)	18 Mos. Fres.: Soya-meal (cwt)	Plow (Acres/Hr)	Disc (Acres/Hr)	Seeder (Acres/Hr)	Roller (Acres/Hr)	Tractor (H.P.)	Feeding System (Animals/Hr)	Combine (Acres/Hr)	Straw Handling System (Acres/Hr)
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
Functional				.81	2.4	-4.48	-.125	0	-.5	2.0	-1.69	-2	-3.5	0	-3.8	-5.32	-.27	-.16	-.165	0	0	0	.19	0	1.0	0	.19	0	0	.19	2.0	.8	.8	.36	.275	.598	9.0	7.4
1	Total Acres	260	≥	6	6																																	
2	Pasture + Silage	0	≥	-1	-3											.56					1			1				1										
3	Barley + Wheat	0	≥	-5	-3	1	1	1																														
4	Stores on Sand (Min)	20	=										1	-1																								
5	Stores on Sand (Max)	50	≥										1																									
6	N	0	≥														10				-6.96	-.45	-.382	-6.96	-1.67	-1.85	-.382	-6.96	-1.85	-.382								
7	P	0	≥															10			-2.80	-.3	-.109	-2.30	-.67	-1.26	-.109	-2.80	-1.26	-.109								
8	K	0	≥																10		-14	-2.24	-.246	-14	-3.6	-1.23	-.246	-14	-1.23	-.246								
9	Stores on Sand SE	147	≥										10.0								-36.9	-4.5	-.715															
10	Stores on Sand DCP	242	≥										9.4								-60.5	-1.79	-4.52															
11	Stores on Sand DM	42.2	=										3.61							-1	-10.5	-4.75	-.117															
12	Stores in Courts SE	0	≥												11.7									-36.9	-6.05	-15.7	-.715											
13	Stores in Courts DCP	0	≥												13.5									-60.5	-8.5	-15.7	-4.52											
14	Stores in Courts DM	0	≥												-2.59									10.5	3.46	2.36	.117											
15	18 Mos. Fres. SE	0	≥													15.0												-36.9	-15.7	-.715								
16	18 Mos. Fres. DCP	0	≥													14.2												-60.5	-15.7	-4.52								
17	18 Mos. Fres. DM	0	≥													-3.62												10.5	2.36	.117								
18	Barley balance	0	≥				-2.0			-1	1	1	.445		.4	.445										1			1									
19	Barley to pigs	195	=									1																										
20	Straw balance	0	≥			-1.3	-1.3	-1	1				1.5		1.5	1.5						1																
21	Straw handling balance	0	≥			-1.3	-1.3	1																														
22	March Disc	0	≥				1																									-26.4						
23	March Seeder	0	≥				1																										-26.4					
24	March Roller	0	≥				1																											-17.6				
25	Tractor	0	≥																												66				-1			
26	August Combine	0	≥				1																														-82	
27	August Bale Hand.	0	≥			1.3		1																														-55.6
28	Sept. Combine	0	≥			1																															-82	
29	Sept. Bale Hand.	0	≥			1.3		1																														-55.6
30	October and November Plow	0	≥			1																									-131							
31	October and November Disc	0	≥			1																										-26.4						
32	October and November Seeder	0	≥			1																											-26.4					
33	October and November Roller	0	≥			1																												-17.6				
34	Feeding	0	≥												1	1																				-4.1		
35	Dec.-Jan. Plow	0	≥				1																								-131							
36	Court space	440	≥												5	5																						
37	Barley - Wheat	0	≥			1	-1																															

SOLUTION TO L-6

ONE-MAN WORKING FORCE

Gross Margin* - £12,380.58

Rotation: G C C C C C (43 acres/break)

Wheat	108 acres
Barley	108 acres
Grazing	16 acres
Silage	27 acres

Livestock:

Stores on sand	50
Feed - 135T Silage 22T Barley	
Stores in courts	59
Feed - 150T Silage 44T Barley	
Fresians	28
Feed - 117T Silage 21T Barley	

Equipment:

Tractor	46 HP (Drawbar)
Plow	4.6 ft.
Seeder	16 ft.
Disc	16 ft.
Roller	21 ft.
Combine	7 - 8 ft. S.P.
Bale Handling System	Farmhand Mechanical
Cattle Feeding	Manual

* Capital-costs of tractor, machinery and livestock feeding equipment, in addition to enterprise variable costs, are deducted from gross revenue.

Table 3.14

The solution shows that the correct acreages were guess in preparing the solution to the machine-sizing problem. The solution to the linear program provides the information that the enterprise combination and scale are optimal considering all program variables including machinery fixed-costs and the daily division of labour between field and farmstead operations. The actual agricultural content of the solution is interesting - in particular, the selection of mechanized bale handling equipment on one hand, and the rejection of mechanized cattle feeding on the other. The "larger than usual" sizes of the equipment might raise some doubts. However, it falls squarely in line with Canadian practice on similar sized farms following similar cropping practices.

Program with a Three-Man Working Force

The alternative to the one-man crew with some help from the farmer, was considered to be a three-man regular work force, with some help from the farmer. This permits potatoes and beets to be included in a program for the farm. Based upon rotations that were acceptable to the farmer, it was guessed that the cropping program would be as follows:

Potatoes	-	37 acres (18 main crop)
Beets	-	37 acres
Grass	-	37 acres
Wheat	-	74 acres
Barley	-	74 acres

The farmer specified that a potato harvester and a beet harvester must be used. As the available range of capacity for these machines is small, the times required for these operations are relatively fixed. The same observations can be made concerning the potato planting operation. The size of a manually fed planter is limited by the crew size. Considering the capacities to be 5 acres per day, $1\frac{1}{2}$ acres per day and 2 acres per day for the potato planter, potato harvester and beet harvester respectively, the times for these operations adjusted for weather become 2 weeks, 3 weeks and 4 weeks respectively. With reference to Fig. 3.16, the remaining critical operations, and times available to perform these operations, were calculated.

Time		Operations		Available Manpower
March 3 weeks	137 hours	Disc and harrow barley and potato land	(111 acres)	3
April 2 weeks	91 hours	Disc and harrow beet land Plant beets	(37 acres) (37 acres)	3
August 4 weeks	156 hours	Combine barley Bale and stack barley straw Plow as much of the barley stubble as possible	(74 acres) (74 acres)	3
September 4 weeks	152 hours	Combine wheat Bale and stack barley straw Plow as much of the barley stubble as possible	(74 acres) (74 acres)	3
Oct. Nov. 4 weeks	152 hours	Plow to requirement for wheat Disc, harrow and seed wheat Feed stores daily	(74 acres) (74 acres)	
Dec. Jan.	156 hours	Plow for barley, potatoes and beets	(148 acres)	

The guiding principles developed in Section 3.2.7.3 were applied to the allocation of operations to each worker as follows:

<u>March</u>

$$\frac{A_B + A_P}{Y_D} \geq 137$$

$$\frac{A_B}{Y_S} \geq 137$$

$$\frac{A_B + A_P}{Y_H} + \frac{\text{Animals x days}}{Y_A} \geq 137$$

<u>April</u>

$$\frac{A_{Be}}{Y_D} \geq 91$$

$$\frac{A_{Be}}{Y_H} + \frac{\text{Animals x days}}{Y_A} \geq 91$$

$$\frac{A_{Be}}{Y_{Be}} \geq 91$$

<u>August</u>

$$\frac{A_B}{Y_C} + \frac{A_B}{Y_B} \geq 152$$

$$\frac{A_B}{Y_{Bale}} + \frac{\text{Acres Plowed}}{Y_P} \geq 156$$

<u>September</u> - Same as August, for wheat

<u>October</u>

$$\frac{A_W}{2Y_P} + \frac{\text{Animals x days}}{2Y_A} \geq 152$$

$$\frac{A_W}{Y_D} + \frac{A_W}{Y_H} + \frac{A_W}{Y_S} \geq 152$$

<u>Dec. Jan.</u>

$$\frac{\text{Remaining acres}}{Y_P} \geq 156$$

Equations were written and solved by Lagrange's method to yield the following values:

$$Y_{Be} = 0.25 \text{ acres/hour}$$

$$Y_C = 0.75$$

$$Y_B = 1.35 \text{ acres/hour}$$

$$Y_P = 0.41 \text{ acres/hour}$$

$$Y_T = 27 \text{ HP}$$

$$Y_A = 10 \text{ animals/hr/man}$$

$$Y_D = 1.3 \text{ acres/hour}$$

$$Y_S = 1.3 \text{ acres/hour}$$

$$Y_H = 1.95 \text{ acres/hour}$$

Times were calculated from these rates of work, and were entered in program L-6 - Three Man Working Force.

The program was solved, but the solution did not agree with the assumption. Cropping was:

Beets - 21 acres

Potatoes - 39 acres (half main crop and half earlies)

Wheat - 159 acres

Barley - 0 acres

The equations for operations were then rewritten as follows:

September $$\frac{159}{Y_C} + \frac{159}{Y_B} \geq 156 \quad (1)$$

$$\frac{159}{Y_{Bale}} + \frac{\text{Acres plowed}}{Y_P} \geq 156 \quad (2)$$

$$\underline{\text{October}} \qquad \frac{138^{*} - \text{Acres plowed in Sept.}}{2Y_P} + \frac{1144}{2Y_A} \geq 152 \qquad (3)$$

$$\frac{138}{Y_D} + \frac{138}{Y_H} + \frac{138}{Y_S} \geq 152 \qquad (4)$$

From (2) -

$$\text{Acres plowed} = \left(156 - \frac{159}{Y_{Bale}}\right) Y_P$$

Substituting in (3) yields:

$$\frac{138}{2Y_P} + \frac{159}{2Y_{Bale}} + \frac{1144}{2Y_A} \geq 230 \qquad (5)$$

Equations (1), (4), and (5) have no common terms, so may be solved individually. Note that all inequalities then become equalities. Using B' values as developed for "L-6 - One-Man Working Force", except for the baler, and the bale handling system, which now work independently, functional equations were respectively written as:

$$90Y_C + 27.2Y_B = C'_{(1)}$$

$$8Y_D + 8Y_S \quad 3.6Y_H = C'_{(4)}$$

$$200^{**}Y_P + 50Y_{Bale} + 5.98Y_A = C'_{(5)}$$

Solving these three pairs of equations yielded:

$Y_P = 0.545$ acres/hour

$Y_D = 2.43$ acres/hour

$Y_S = 2.43$ acres/hour

$Y_H = 3.60$ acres/hour

* 159 - Beet acreage. It was considered that the 21 acres of wheat on beet land would have to be planted towards spring.

** Arises by combining with a tractor qualification, as in "L-6 - One-Man Working Force".

Y_A = 9* animals/hour

Y_C = 1.58 acres/hour

Y_B = 2.86 acres/hour

Y_{Bale} = 1.70 acres/hour

Y_T = 36 HP

From these calculated capacities, the following critical times were calculated:

Discing in April	137 hours
Seeding in April	137 hours
Harrowing in April	115 hours
Beet seeding in April	91 hours
Combining in August or September	100 hours
Bale handling in August or September	56 hours
Plowing in August or September	63 hours
Plowing in October	95 hours
Discing in October	57 hours
Seeding in October	57 hours
Harrowing in October	38 hours
Animal feeding	4.4 hours/day

These times were entered in the "L-6 - Three-Man Working Force" matrix.

* As 20 animals per hour represents reasonable hand feeding practice, this figure was used in the program.

The summary of results of program L-6, 3-man working force, is shown in Table 3.15.

The implement sizes, as determined by the program, are equal to the optimal combination calculated by minimizing the fixed-cost function by the method of Lagrange multipliers. It should be noted that in this case two complete iterations were required to obtain the solution. The program provides a solution that has selected enterprises with due regard for the fixed-cost of machines at the same time that a combination of machines was selected with due regard for enterprise combination and scale.

The gross-margin of £14,409 exceeds that of the same farm operated by one man by £(14,409 - 12,380)

= £ 2,020

This must cover the yearly fixed-cost of two additional tractors and the yearly wages of two additional men. It is apparent that these costs can be covered with a small margin remaining. The farm size, however, is approaching the minimum size in which a "regular" crew can be justified.

THREE-MAN WORKING FORCE

Constraint No.	Requirement (Name)	Requirement (Number)	Inequality sign	G G C P C Be C (1)	G C C P C Be C (2)	G C C P R C Be_R (3)	C_3 (Acres) (4)	Be_3 (Acres) (5)	W (Acres) (6)	B (Acres) (7)	Barley to Pigs (T) (8)	Barley Bought (T) (9)	Early Potatoes (Acres) (10)	Main Crop (Acres) (11)	New Earlies (Acres) (12)	Straw Handled (T) (13)	Straw Sold (T) (14)	Stores on Sand (15)	Stores on Sand (Artificial) (16)	Stores in Courts (17)	18 Mos. Fresians (18)	N (Units X 10^{-2}) (19)	P (Units X 10^{-2}) (20)
Functional				4.62	4.62	0	0	4.62	4.48	.125	2	-2	7.5	7.49	7.1	0	.5	3.5	0	3.8	5.32	.27	.16
1	Total Acres	260	≥	7	7	6																	
2	C_3 + Beets	0	≥			-1	1	1															
3	Pasture + Silage	0	≥	-2	-1	-1															.56		
4	Barley + Wheat	0	≥	-3	-4	-3	-1		1	1													
5	Stores on Sand (Min)	20	=															1	-1				
6	Stores on Sand (Max)	50	≥															1					
7	N	0	≥																			10	
8	P	0	≥																				10
9	K	0	≥																				
10	Mangles	0	≥	-3	-3	-3		-3															
11	Beet Tops	0	≥	-1.2	-1.2			-1.2															
12	Stores on Sand: SE	147	≥															10					
13	Stores on Sand: DCP	242	≥															9.4					
14	Stores on Sand: DM	42.2	=															3.61					
15	Stores on Sand: Beet Tops (lim)	0	≥															-1.3					
16	Stores: SE	0	≥																	11.7			
17	Stores: DCP	0	≥																	13.5			
18	Stores: DM	0	≥																	-2.59			
19	Stores: Beet Tops (lim)	0	≥																	-1.3			
20	18 Mos. Fres.: SE	0	≥																		15		
21	18 Mos. Fres.: DCP	0	≥																		14.2		
22	18 Mos. Fres.: DM	0	≥																		-3.62		
23	18 Mos. Fres.: Beet Tops (lim)	0	≥																		-1.3		
24	Barley balance	0	≥							-2	1	-1						.445		.4	.445		
25	Barley to pigs	195	=								1												
26	Straw balance	0	≥													-1	1	1.5		1.5	1.5		
27	Straw handling balance	0	≥						-1.3	-1.3						1							
28	March: Disc pot. & barley	0	≥	1	1	1				1													
29	March: Harrow pot. & barley	0	≥	1	1	1				1													
30	March: Seed barley	0	≥							1													
31	March: Plant potatoes	0	≥	1	1	1																	
32	March: Feed stores	0	≥																	1	1		
33	April: Disc beets	0	≥		1			1															
34	April: Harrow beets	0	≥																				
35	April: Seed beets	0	≥		1			1															
36	Sept. & Aug.: Combine barley	0	≥							1													
37	Sept. & Aug.: Bale handling	0	≥							1													
38	Sept. & Aug.: Combine wheat	0	≥						1														
39	Sept. & Aug.: Bale handling	0	≥						1														
40	Harvest potatoes	0	≥											1									
41	Plow B, Be, and P	0	≥	2	2	1		1		1													
42	Disc W - Be	0	≥	-1	-1			-1	1														
43	Harrow W - Be	0	≥	-1	-1			-1	1														
44	Seed W - Be	0	≥	-1	-1			-1	1														
45	Lift beets	0	≥		1			1															
46	Plow W - Be	0	≥	-1	-1			-1	1														
47	Courts	440	≥																	5	5		
48	Tractor	0	≥																				
49	Potatoes	0	≥	-1	-1	-.86							1	1	1								
50	Potatoes	0	≥										-1	1									

Constraint No.	Requirement (Name)	K (Units X 10^{-2}) (21)	DM (Artificial) (22)	Stores on Sand: Mangles (10T) (23)	Stores on Sand: Beet Tops (10T) (24)	Stores on Sand: Silage (15T) (25)	Stores on Sand: Straw (T) (26)	Stores on Sand: Soya-meal (cwt) (27)	Stores in Courts: Mangles (10T) (28)	Stores in Courts: Beet Tops (10T) (29)	Stores in Courts: Silage (15T) (30)	Stores in Courts: Hay Purchased (T) (31)	Stores in Courts: Barley (T) (32)	Stores in Courts: Soya-meal (cwt) (33)	18 Mos. Fres.: Mangles (10T) (34)	18 Mos. Fres.: Beet Tops (10T) (35)	18 Mos. Fres.: Silage (15T) (36)	18 Mos. Fres.: Barley (T) (37)	18 Mos. Fres.: Soya-meal (cwt) (38)	Plow (Acres/Hr) (39)	Disc (Acres/Hr) (40)
Functional		.165	0	0	0	0	0	-.19	0	0	0	-1.0	0	-.19	0	0	0	0	-.19	-2.0	-.8
1	Total Acres																				
2	C_3 + Beets																				
3	Pasture + Silage					1					1						1				
4	Barley + Wheat																				
5	Stores on Sand (Min)																				
6	Stores on Sand (Max)																				
7	N			-2.46	-5.56	-6.96	-.45	-.382	-2.46	-5.56	-6.96	-1.67	-1.85	-.382	-2.46	-5.56	-6.96	-1.85	-.382		
8	P			-1.18	-1.87	-2.80	-.3	-.109	-1.18	-1.87	-2.80	-.67	-1.26	-.109	-1.18	-1.87	-2.80	-1.26	-.109		
9	K	10		-9	-9.3	-14	-2.24	-.246	-9	-9.3	-14	-3.6	-1.23	-.246	-9	-9.3	-14	-1.23	-.246		
10	Mangles			10					10						10						
11	Beet Tops				1					1						1					
12	Stores on Sand: SE			-15.6	-20	-36.9	-4.5	-.715													
13	Stores on Sand: DCP			-13.4	-44.8	-60.5	-1.79	-4.52													
14	Stores on Sand: DM		-1	-2.47	-4.5	-10.5	-4.75	-.117													
15	Stores on Sand: Beet Tops (lim)				10																
16	Stores: SE								-15.6	-20	-36.9	-6.05	-15.7	-.715							
17	Stores: DCP								-13.4	-44.8	-60.5	-8.5	-15.7	-4.52							
18	Stores: DM								2.47	4.5	10.5	3.46	2.36	.117							
19	Stores: Beet Tops (lim)									10											
20	18 Mos. Fres.: SE														-15.6	-20	-36.9	-15.7	-.715		
21	18 Mos. Fres.: DCP														-13.4	-44.8	-60.5	-15.7	-4.52		
22	18 Mos. Fres.: DM														2.47	4.5	10.5	2.36	.117		
23	18 Mos. Fres.: Beet Tops (lim)															10					
24	Barley balance												1					1			
25	Barley to pigs																				
26	Straw balance						1														
27	Straw handling balance																				
28	March: Disc pot. & barley																				-137
29	March: Harrow pot. & barley																				
30	March: Seed barley																				
31	March: Plant potatoes																				
32	March: Feed stores																				
33	April: Disc beets																				-91
34	April: Harrow beets																				
35	April: Seed beets																				
36	Sept. & Aug.: Combine barley																				
37	Sept. & Aug.: Bale handling																				
38	Sept. & Aug.: Combine wheat																				
39	Sept. & Aug.: Bale handling																				
40	Harvest potatoes																				
41	Plow B, Be, and P																			-180	
42	Disc W - Be																				-57
43	Harrow W - Be																				
44	Seed W - Be																				
45	Lift beets																				
46	Plow W - Be																			-253	
47	Courts																				
48	Tractor																			66	
49	Potatoes																				
50	Potatoes																				

THREE-MAN WORKING FORCE

	Stores on Sand					Stores in Courts						18 Mos. Fres.																
DM (Artificial)	Mangles (10T)	Beet Tops (10T)	Silage (15T)	Straw (T)	Soya-meal (cwt)	Mangles (10T)	Beet Tops (10T)	Silage (15T)	Hay Purchased (T)	Barley (T)	Soya-meal (cwt)	Mangles (10T)	Beet Tops (10T)	Silage (15T)	Barley (T)	Soya-meal (cwt)	Plow (Acres/Hr)	Disc (Acres/Hr)	Seeder (Acres/Hr)	Harrow (Acres/Hr)	Tractor (H.P.)	Feeding System (Animals/Hr)	Combine (Acres/Hr)	Straw Handling (Acres/Hr)	Potato Planting (Acres/Hr)	Potato Harvester (Acres/Hr)	Beet Planter (Acres/Hr)	Beet Harvester (Acres/Hr)
22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
0	0	0	0	0	-.19	0	0	0	-1.0	0	-.19	0	0	0	0	-.19	-2.0	-.8	-.8	-.36	-.275	-.598	-9.0	-2.72	-29.0	-78.0	-12.0	-20.8
			1					1						1														
	-2.46	-5.56	-6.96	-.45	-.382	-2.46	-5.56	-6.96	-1.67	-1.85	-.382	-2.46	-5.56	-6.96	-1.85	-.382												
	-1.18	-1.87	-2.80	-.3	-.109	-1.18	-1.87	-2.80	-.67	-1.26	-.109	-1.18	-1.87	-2.80	-1.26	-.109												
	-9	-9.3	-14	-2.24	-.246	-9	-9.3	-14	-3.6	-1.23	-.246	-9	-9.3	-14	-1.23	-.246												
	10					10						10																
		1					1						1															
	-15.6	-20	-36.9	-4.5	-.715																							
	-13.4	-44.8	-60.5	-1.79	-4.52																							
-1	-2.47	-4.5	-10.5	-4.75	-.117																							
		10																										
						-15.6	-20	-36.9	-6.05	-15.7	-.715																	
						-13.4	-44.8	-60.5	-8.5	-15.7	-4.52																	
						2.47	4.5	10.5	3.46	2.36	.117																	
							10																					
												-15.6	-20	-36.9	-15.7	-.715												
												-13.4	-44.8	-60.5	-15.7	-4.52												
												2.47	4.5	10.5	2.36	.117												
													10															
										1					1													
				1																								
																		-137										
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																		-91										
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																				-38								
																			-57									
																												-156
																	-253											
																	66				-1							

SOLUTION TO L-6

THREE-MAN WORKING FORCE

Gross Margin* - £ 14,409.35

Rotation: G W W P C Be W (21 acres/break)

G W W P_R W W (18.6 acres/break)

Wheat	159 acres
Barley	0 acres
Grazing	19 acres
Silage	21 acres
Potatoes	37 acres
Beets (and mangles)	21 acres
Roots	2.6 acres

Livestock:

Stores on land		50
Feed -	Silage	90 tons
	Beet tops	65 tons
	Barley	22 tons
Stores in courts		54
Feed -	Silage	100 tons
	Beet tops	70 tons
	Mangles	4 tons
	Barley	34 tons
Fresians (18 months)		34
Feed -	Silage	117 tons
	Beet tops	44 tons
	Mangles	85 tons
	Barley	15 tons

Equipment:

Tractor	36 H.P. (drawbar)
Plow	3 ft.
Disc	10 ft.
Seeder	10 ft.
Harrow	13 ft.
Combine	$8\frac{1}{2}$ - 10 ft. SP
Bale handling system	Farmhand
Cattle feeding	Manual
Potato planter	2 row
Beet planter	2 row
Beet harvester	1 row

* Capital costs of 1 tractor, machinery and livestock feeding equipment, in addition to enterprise variable costs, are deducted from gross revenue. 2 additional small tractors are required for this program.

Table 3.16

4. CONCLUSIONS

4.1 LINEAR PROGRAMMING AS AN AID TO FARM PLANNING IN THE LOTHIANS

The variety of problems presented by the six case studies is generally representative of arable farming in the Lothians of Scotland. All of the problems that were presented were "fielded" and a method was, in each case, found that permitted the problem to be included in the linear program. This fact, coupled with results that in each case were considered to be reasonable, offers substantial proof that the method can represent arable agriculture in the Lothians.

Linear programming builds up solutions for individual farms to a large extent from physical and biological data. The method develops the full potential permitted by the system of restraints imposed on it, rather than by measuring the potential against statistically determined "bench-mark" practices for the area. The fact that the solutions obtained by linear programming agreed closely with statistically determined practices for the area is considered to offer further proof that the method is capable of providing good results. (The stable conditions that have permitted a statistical determination of bench-mark practices might appear to reduce the need for an analytic technique such as linear programming. However, even under these conditions linear programming can be useful in emphasizing the value of bench-mark practices.)

The gross margin technique has been accepted as a useful aid to farm planning in the East of Scotland. The fact that the gross margin technique requires identical input information, coupled with the fact that the gross margin method is not remedial in nature, but requires some additional optimizing technique, should lead to the inescapable conclusion that a linear program should complement each gross margin analysis.

The practicability of the method, however, is not proven by either a demonstration that it can produce good results, or by advancing the corolary that acceptance of the gross margin technique implies acceptance of linear programming, or even by the paradox that linear programming, under certain conditions, is not necessary but should be used. The practicability depends to a large measure on other factors. Experience gained in the preparation of these programs gives rise to views that might be considered in association with other views on this subject referred to in the literature review. There are, in total, comparatively few people actively engaged in practical farm linear programming. The reason may be two-fold.

First, a formidable hurdle must be surmounted in getting past the threshold of linear programming. Whether this hurdle is in truth as formidable as it appears, or whether it exists largely because of the failure to develop a suitable training method, poses a question that deserves investigation.

Second, those who have met the challenge and acquired the skills of practical programming appear to lose interest after a short time. This may be because of repeatedly facing the demanding and time-consuming task of preparing detailed individual programs. This practical, but vital, aspect of linear programming has been neglected in the rush to extend the frontier of feasible applications. The principle of decomposition formulated by Dantzig should be of particular interest to those who are engaged in preparing farm linear programs. The application of this principle reduces the preparation of a program for an individual farm to selecting applicable prepared part-programs and then preparing a small linking matrix. The development of this technique could bring about a major breakthrough, in many ways analgous to that brought about by compilers in simplifying digital computer programming.

4.2 AVAILABILITY AND SUITABILITY OF SCOTTISH AGRICULTURAL RESEARCH DATA

A considerable number of data are required for the preparation of comprehensive farm programs. As indicated throughout Section 3.1, most of the data appearing in the programs were obtained from published research results of the Edinburgh School of Agriculture, personal interviews with members of the staff, and the records of the co-operating farmers. Remaining data were obtained from publications readily available in the United Kingdom.

The data for soil fertility, livestock feeding, grazing, and silage making resulted from experimentation prompted primarily by biological considerations (and certainly without any concern for applicability to linear programming). The results, as presented, were well suited to inclusion in linear programming. It is of particular interest to note that the research that led to the New Method of feed formulation was of such a basic nature that, at its inception, any prospect of easily adaptable economic data resulting from the experiment must have appeared unlikely. It now is seen that in this case the search for fundamental biological truths did not preclude the development of useful economic relationships. In fact, it is unlikely that any experiment designed with the specific aim of developing production functions could have provided as easily adaptable and universal relationships as those arising out of this basic study.

There appeared to be a complete lack of data relating farming operations to the weather. This deficiency to a considerable extent makes the machine-size calculations in L-6 suspect, and raises a question concerning the weather-dependent labour requirements in the other programs. It is of interest to note that the weather data developed as indicated from the Climatological Atlàs of the British Isles were used to check a series of gross margin analyses carried out during a "former students" short course at the Edinburgh School of Agriculture. These calculations agreed closely with labour standards as determined by the account analysis procedure. As a result, it is felt that the values appearing in the programs can be used with some confidence until a research effort is made to describe weather in terms of distribution functions that are related to specific weather-dependent activities.

Standard references on machinery operating characteristics provided the source of most of the data required to prepare the equipment selection section of program L-6. The absence of draft (or power) requirements in these references was a notable omission. These data were obtained from an American source. It would appear that data relating capacity to implement size were quite coarse. If the method of equipment selection proposed in this thesis should be developed for general use (or, in fact, if machinery selection is to be based upon any method using more precise weather data) a breakdown between operating times, avoidable and unavoidable delays, would be of great value. Industry, in using the term "duty" of machines, implies an appreciation of the cost of idle time in terms of alternative capital investment. More precise time studies are needed before this measure has an economic meaning in agriculture.

4.3 LINEAR PROGRAMMING IN THE ANALYSIS OF MECHANIZED ACTIVITIES

For purposes of this thesis, items of the farm plant were defined as structures, materials handling and series machines. The program served to illustrate, through practical example of removating buildings, building silos, adding feed distribution equipment, and finally, simultaneously selecting a feeding system and a full line of field equipment, that mechanized activities can be analysed in full economic association with other farm activities.

The area of broadest interest should be in the selection of a line of field or series equipment, as this is the most general classification, and the area in which this thesis, insofar as reading on the subject can determine, would appear to contribute most to the advancement of knowledge. The method of combining Lagrange's multipliers with linear programming to simultaneously select equipment and farm enterprises is not perfectly developed in this thesis. First, the method of Lagrange's multipliers will not in every case provide a solution, and second, the allocation of tasks between workers of a multiple-man working force was left as an arbitrary exercise. The actual application to program L-6, however, indicated that both of these problems may be largely of academic interest. The equation solved readily by Lagrange's method, and the optimum allocation of tasks, appeared quite obvious. The ease with which L-6 was formulated and solved might even suggest that the generalizations developed in the series of examples as "guiding principles" might be superfluous. It is hoped, on the other hand, that these principles might be considered useful for less rigorous analyses.

4.4 MECHANIZATION FOR BEEF CATTLE FEEDING

The six farms included a wide variety of active cattle enterprises amongst their activities. When the active enterprises were added to acceptable alternatives, a comprehension range of possibilities was available for study.

The results of the linear programs show that under the price relationships that prevailed, cattle feeding paid on every farm. (This must be qualified by noting that capital for the purchase of store cattle was available on a non-competitive basis on all farms.) The pattern of feeding enterprises selected for any particular farm depended upon the restraints imposed. The wide variety of restraints makes any generalization difficult. However, in general, barley beef animals were extremely competitive, and suckler cows were equally uncompetitive. The use of the SE system of ration formulation does not treat animals at a low level of production fairly. For this reason, the generalization on cows should be treated with caution. The New Method would have been more revealing.

The farms permitted quite a broad study of cattle feeding mechanization. The absence of a commercial feedlot provides a notable exception. The economics of commercial feedlots are of such a nature that the optimum level of mechanization can often be determined by the application of differential calculus to a single equation. The fact that feedlots do lead the way in advanced techniques, and thereby provide "good copy", unquestionably influences the entire feeding industry. The converse is not true. This study of farm cattle feeding has little application to the feedlot industry.

Within the patterns of farming studied, and with the price structure that prevailed, the programs revealed two specific aspects of the mechanization of beef cattle feeding:

1. If grass is considered as a requirement in the rotation, complete mechanization of forage handling <u>can</u> (but not necessarily <u>will</u>) be justified.

2. The saving in nutritive value does not justify the replacement of horizontal silos with sealed tower types.

This study has presented the hypothesis that beef cattle feeding should be considered as competing for time with other farm activities (principally spring and autumn field work). As such, the relative levels of mechanization are dependent on relative scale of enterprises, and relative fixed-cost per unit of mechanization between field and farmstead.

It was shown in program L-6 that the present relative fixed-costs are such that, even under the competitive pressures developed by one-man operation of a 260-acre farm (without row crops), a 90-head feeder enterprise should be a hand-feeding operation. This and other results generated in precisely the same manner can serve as bench-marks from which general relationships can be developed. It can be readily seen from illustrations of Lagrange's method that the effect of scale on relative levels of mechanization between competing enterprises is determined by the factor

$$\sqrt{\frac{\text{Scale, proposed}}{\text{Scale, bench-mark}}}$$

Related to the results of L-6, doubling the cattle enterprises would have altered the times devoted to feeding and field operations by the factor 1.41, and, of course, implies a degree of mechanization in the feeding method.

It is suggested that a study of the economics of farm beef production vs. feedlot beef production, using the criteria developed here for determining the optimum level of mechanization for farm beef, would constitute a very useful exercise in providing background data for government policy.

The working force picture in the Lothians is in dramatic contrast with that in Alberta. This thesis has shown this to be a result of the dominant place of potatoes in the pattern of agriculture in the Lothians and the absence of high-capacity potato harvesters. The effect of the present large labour force is to maintain the level of mechanization of other farm enterprises, including beef cattle feeding, lower than the Alberta optimum. This is clearly illustrated in L-6 where the optimum level of feedlot mechanization was halved by the addition of two men to the farm staff. The extent of the influence of the bottleneck imposed by the absence of high-capacity two-man potato harvesters has been emphasized at every turn in the preparation of this thesis. It is a tribute to the inherent value of linear programming, entirely appropriate to the Lothians, that linear programming can tie the future level of beef feeding mechanization directly to future developments in potato handling.

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APPENDIX A

ENERGY REQUIREMENTS*

A. Energy for Maintenance

It is assumed that:

(a) The maintenance requirement of older animals is 90 kcal N.E. per kg L.W. $^{0.73}$ and that of younger animals, 100 kcal. (Categories 1 and 5 are considered to be older animals.)

(b) 1 lb. S.E. supplies 1410 kcal N.E. for maintenance.

Category of Animal	Live-weight (lb.)	(kg.)	($kg^{0.73}$)	Requirement (kcal N.E.)	(lb. S.E.)
1	1,100	500	93.4	8,400	6.0
2	750	340	70.5	7,050	5.0
3	700	320	67.4	6,740	4.8
4	575	260	57.9	5,790	4.1
5	950	430	83.7	7,530	5.3

B. Energy for Production

(i) Milk. The cows should yield perhaps 2 gallons per day. One gallon contains 3,400 kcal and 1 lb. S.E. supplies 1,290 kcal N.E. for milk production. The total requirement will therefore be

$$\frac{3,400}{1,290} \times 2 \quad 5.3 \text{ lb. S.E.}$$

(ii) Live-weight gain. It is assumed that the energy values of 1 lb. increments are as shown below, and that 1 lb. S.E. supplies 1,071 kcal N.E. for live-weight gain.

* Prepared by J.F.D. Greenhaugh, Edinburgh School of Agriculture, 1963.

Category of Animal	Daily live-weight gain (lb.)	(kg.)	kcal/kg. gain	Requirement (kcal N.E.)	(lb. S.E.)
2 (a)	1.8	0.8	4,000	3,200	3.0
(b)	2.3	1.0	4,200	4,200	3.9
3 (a)	1.6	0.7	3,800	2,700	2.5
(b)	2.0	0.9	4,000	3,600	3.4
4	1.25	0.6	3,300	2,000	1.9
5	1.5	0.7	4,800	3,400	3.2

C. Total Energy Requirements

Category of Animal	Requirement/day (lb. S.E.)
1	11.3
2 (a)	8.0
(b)	8.9
3 (a)	7.3
(b)	8.2
4	6.0
5	8.5

Maximum dry matter intake per day

This will vary with the average digestibility of the organic matter of the ration (calculated as the weighted mean of the digestibility values for the constituents). It is estimated that in the ranges of organic matter digestibility 50 - 60, 60 - 70, and 70 - 80% the daily intakes of dry matter by the non-lactating animals will be 7.0, 8.2 and 9.5% respectively, of metabolic L.W. The cows should eat 20% more than this.

Category of Animal	Digestibility Range (%)		
	50-60	60-70	70-80
1	17.3	20.3	23.4
2	10.9	12.7	14.8
3	10.5	12.2	14.1
4	8.9	10.5	12.1
5	12.9	15.1	17.5

Protein Requirements

These have been calculated factorially, using the following assumptions:

(a) Endogenous urinary N 2 mg. per kcal. fasting metabolism.

(b) Protein contents of L.W. gains are 16% for categories 2 and 3, 20% for 4, and 12% for 5. Milk contains 160 g. protein per gallon.

(c) Dry matter intake is 0.08 kg. per kg. L.W.$^{0.73}$ and metabolic faecal protein excretion 28.8 g. per kg. dry matter intake.

(d) The biological value of ruminants' food protein is 60% for young stock and 70% for milking cows.

1	2	3	4	5	6	7	8	9
Category of Animal	L.W. (kg.$^{0.73}$)	Endogenous N × 6.25 (g)	Protein Stored (g)	Net Requirement (g protein)	Available Requirement (g protein)	Metabolic Faecal Protein g	g × $\frac{40}{100}$	D.C.P. Requirement (lb)
1	93.4	105	320	425	607	215	64	1.48
2 (a)	70.5	88	128	216	360	162	65	0.94
(b)	70.5	88	160	248	413	162	65	1.05
3 (a)	67.4	84	112	190	317	155	62	0.83
(b)	67.4	84	144	228	380	155	62	0.97
4	57.9	72	120	192	320	133	53	0.82
5	83.7	94	84	178	297	193	77	0.82

g × $\frac{30}{100}$

APPENDIX B

SOME CALCULATIONS OF FEEDING VALUES

USING THE NEW METHOD

Metabolizable energy for oats	-	1135 kcal/lb as fed
Metabolizable energy for hay	-	900 kcal/lb as fed
Dry matter oats	-	88% as fed
Dry matter hay	-	85% as fed

Example 1:

Determine the pounds of feed required for an animal on maintenance at 6000 kcal/day.

(a) All Hay

$$\frac{m}{d} = \frac{900}{.85} = 1059 \text{ kcal/lb}$$

$$\eta_m = 54.3 + (0.0143 \times 1059)$$

$$= 69.44\%$$

$$M = \frac{6000}{.6944} = 8640.5 \text{ kcal/day}$$

$$M' = M = 8640.5 \text{ kcal/day}$$

$$= \frac{8640.5}{900}$$

$$= \underline{9.601 \text{ lb/day}}$$

(b) All Oats

$$\frac{m}{d} = \frac{1135}{.88} = 1290 \text{ kcal/lb}$$

$$\eta_m = 54.3 + (0.0143 \times 1290)$$

$$= 72.75\%$$

$$M = \frac{6000}{0.7275} = 8247 \text{ kcal}$$

$$M' = M = 8247 \text{ kcal}$$

Oats required

$$= \frac{8247}{1135}$$

$$= \underline{7.266 \text{ lb/day}}$$

(c) 25% Oats

Using proportions:

Energy supplied by Oats = 25% x 8247 = 2062 kcal/day

Energy supplied by Hay = 75% x 8641 = 6481 kcal/day

Total energy = $\underline{8543 \text{ kcal/day}}$

Checking by direct solution:

$$\frac{M}{D} = \frac{8543}{(.88 \times 1.815) + (.85 \times 7.21)}$$

$$= 1105$$

$$\eta_m = 54.3 + (0.0143 \times 1105)$$

$$= 70.1\%$$

$$M' = M = \frac{6000}{0.701} = 8559 \text{ kcal/day}$$

Error = (8559 - 8543) = -16 kcal/day

(d) 50% Oats

Using proportions:

Oats = 3.633 lb; 4123 kcal/day

Hay = 4.800 lb; 4320 kcal/day

Total = 8443 kcal/day

Checking by direct solution as in 1. (c),

Error = -19 kcal/day

= 0.21%

(e) 75% Oats

Using proportions:

Oats = 5.449 lb; 6185 kcal/day

Hay = 2.400 lb; 2160 kcal/day

Total = 8345 kcal/day

Checking by direct solution as in 1. (c),

Error = -16 kcal/day

= 0.19%

Example 2:

Determine the pounds of feed required for an animal requiring 6000 kcal of energy for maintenance and 3000 kcal of energy for fat deposition:

(a) All Hay

$\frac{m}{d}$ = 1059 kcal/lb.$_{dm.}$ (from Example 1)

η_m = 69.44% (from Example 1)

η_p = 3 + (0.405 x 1059)

= 45.89%

Energy for maintenance $= \frac{6000}{.6944} =$ 8641 kcal/day

Energy for fat deposition $= \frac{3000}{.4589} =$ 6537 kcal/day

Total = M = 15178 kcal/day

$$M^1 = \frac{15,178}{1 \div 0.11 \quad \frac{M^1}{8641} - 1 \quad - \quad \frac{190 \quad \frac{M^1}{8641} - 1}{1059}}$$

$$= \underline{16,155 \text{ kcal/day}}$$

$$\text{Hay required} = \frac{16,155}{900}$$

$$= 17.95 \text{ lb/day}$$

(b) All Oats

$\frac{m}{d} = 1290 \text{ kcal/lb.}_{dm.}$ (from Example 1)

$\eta_m = 72.75\%$ (from Example 1)

$$\eta_p = 3 + (0.0405 \times 1290)$$

$$= 55.25\%$$

$$\text{Energy for maintenance} = \frac{6000}{0.7255} = 8247 \text{ kcal/day}$$

$$\text{Energy for deposition} = \frac{3000}{0.5525} = 5430 \text{ kcal/day}$$

$$\text{Total} = M = 13677 \text{ kcal/day}$$

$$M^1 = \frac{13,677}{1 + 0.11 \quad \frac{M^1}{8247} - 1 \quad - \quad \frac{190 \quad \frac{M^1}{8247} - 1}{1290}}$$

$$= \underline{14,050 \text{ kcal/day}}$$

Oats required $= \dfrac{14{,}050}{1135}$

$= \underline{12.38 \text{ lb/day}}$

(c) 25% Oats

Using proportions as in Example 1,

Oats = 3.095 lb; 3,512 kcal/day

Hay = 13.463 lb; 12,116 kcal/day

Total = 15,628 kcal/day

Checking by direct solution

$$\frac{M}{D} = \frac{15{,}628}{(.88 \times 3.095) + (.85 \times 13.463)}$$

$$= 1103 \text{ kcal/lb}_{\text{am.}}$$

$$\eta_m = 54.3 + (0.143 \times 1103) \qquad 70.07\%$$

$$\eta_p = 3 + (0.405 \times 1103) \qquad 47.67\%$$

Energy for maintenance $= \dfrac{6000}{0.7007} = 8{,}563$ kcal/day

Energy for fat deposition $= \dfrac{3000}{0.4767} = 6{,}293$ kcal/day

Total = M = 14,856 kcal/day

$$M^1 = \frac{14{,}856}{1 + 0.11 \quad \dfrac{M^1}{8563} - 1 \quad - \quad \dfrac{190 \dfrac{M^1}{8563} - 1}{1103}}$$

$$= \underline{15{,}660 \text{ kcal/day}}$$

Error = (15,660 - 15,628)

= -32 kcal = 0.20%

(d) 50% Oats

Using proportions

Oats = 6.19 lb; 7,025 kcal/day

Hay = 8.97 lb; 8,078 kcal/day

Total = 15,103 kcal/day

Error = -42 kcal/day

= 0.28%

(e) 75% Oats

Using proportions

Oats = 9.285 lb; 10,538 kcal/day

Hay = 4.488 lb; 4,039 kcal/day

Total = 14,577 kcal/day

Error = -35 kcal/day

= 0.24%

Example 3:

Determine the pounds of feed required for an animal requiring 6000 kcal of energy for maintenance and 6000 kcal of energy for fat deposition.

(a) All Hay

$\frac{m}{d}$ = 1059 kcal/$lb._{am.}$ (from Example 1)

η_m = 69.44% (from Example 1)

η_p = 45.89% (from Example 1)

Energy for maintenance = 8,641 kcal/day (from Example 1)

Energy for fat deposition $= \frac{6000}{0.4589} =$ 13,075 kcal/day

Total = M = 21,715 kcal/day

$$M^1 = \frac{21,715}{1 + 0.11 \left(\frac{M^1}{8641} - 1\right) - \frac{190\left(\frac{M^1}{8641} - 1\right)}{1059}}$$

$= \underline{25,000 \text{ kcal/day}}$

Hay required $= \frac{25,000}{900}$

$= \underline{27.78 \text{ lb/day}}$

(b) All Oats

$\frac{m}{d}$ = 1290 kcal/lb.$_{am.}$ (from Example 1)

η_m = 72.75% (from Example 1)

η_p = 55.25% (from Example 2)

Energy for maintenance $= \frac{6000}{0.7275} =$ 8,247 kcal/day

Energy for fat deposition $= \frac{6000}{0.5525} =$ 10,860 kcal/day

Total = M = 19,107 kcal/day

$$M^1 = \frac{19{,}107}{1 + 0.11 \left(\frac{M^1}{8247} - 1\right) - \frac{190\left(\frac{M^1}{8247} - 1\right)}{1290}}$$

$= \underline{20{,}200 \text{ kcal/day}}$

Oats required $= \frac{20{,}200}{1135}$

$= 17.80$ lb/day

(c) 50% Oats

By proportion:

Oats = 8.90 lb; 10,100 kcal/day

Hay = 13.89 lb; 12,500 kcal/day

Total = 22,600 kcal/day

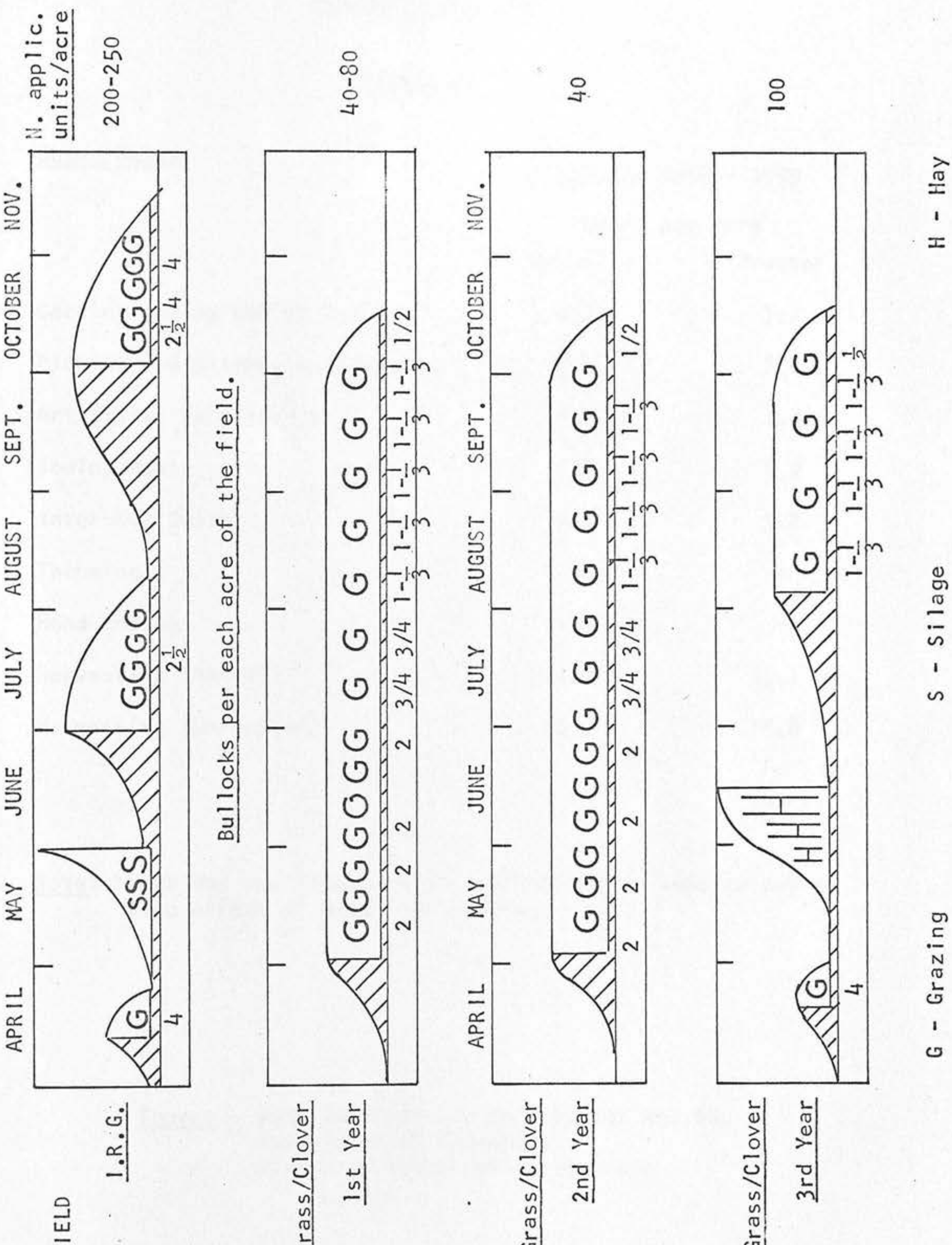
By direct solution as in Example 1. (c) and 2. (c).

Error = 24 kcal/day

= 0.11%

APPENDIX C

GRAZING SYSTEM FOR FATTENING BULLOCKS*



* Holmes, J.C., "Grassland Management for Beet Production". The Production and Use of Grass, 1962. The Edinburgh School of Agriculture.

APPENDIX D

STANDARD TIME DATA

Table D1

Sugar Beets

	Average 1958 - 1959	
	Hours per Acre	
	Manual	Tractor
Carting and Spreading F.Y.M.	6.2	3.7
Plowing and primary cultivation	8.4	7.6
Artificial fertilizers	1.4	1.0
Sowing Beets	1.3	1.0
Inter-Row Cults	6.9	3.2
Thinning	27.0	-
Hand Hoeing	16.8	-
Harvesting (Manual)	61.6	13.1
Harvesting (Harvester)	30.9	15.8

Note: High and low figures also available from same survey - also effect of precision seeding.

Source: Compiled from Economic Report No. 64, Department of Economics, Edinburgh School of Agriculture.

Table D2

Potatoes

	1957 Hours per Acre	
	Manual	Tractor
Carting and Spreading F.Y.M.*	8.0	5.0
Plowing	2.9	2.9
Primary Cultivations	2.5	2.4
Planting - Hand	17.4	2.7
Planting - Planter	9.3	3.6
Inter-Row Cultivations	4.6	4.6
Hand Hoeing	7.4**	-
Spraying	0.8	0.7
Lifting and Pitting	71.1	18.2
Dressing	28.9	-

Required - Harvester data

* 10T/Acre

** Only on 10 farms

Source: Compiled from Economic Report No. 53,
Department of Economics,
Edinburgh School of Agriculture.

Table D3

Barley

	Average 1960 - 1961	
	Hours per Acre	
	Manual	Tractor
Plowing	2.3	2.3
Cultivating	.3	.3
Harrowing	.7	.7
Rolling	.3	.3
Drilling and Fertilizing	.9	.6
Spraying	.2	.2
Combining	1.0	-
Baling	.8	.6
Carting	3.1	1.5
Drying	0.4	-

Source: Compiled from Economic Report No. 77,
Department of Economics,
Edinburgh School of Agriculture.

Table D4

Winter Wheat

	1962	
	Hours per Acre	
	Manual	Tractor
Plowing	2.3	2.3
Discing	.8	.8
Harrowing	.8	.8
Rolling	.2	.2
Drilling	.7*	.7
Fertilizing	.6	.6
Spraying	-	-
Combining	1.1	-
Baling	.8	.8
Carting	1.6	-

* One man only considered.

Source: Compiled from Unpublished Economic Survey,
Department of Economics,
Edinburgh School of Agriculture.

Table D5

Store Cattle in Conventional Courts - Hand Feeding

		Min/day/animal
Feeding and bedding		
(adjusted for timeliness of operation)		
Average of 7 -		3.62
Best	1 -	2.20
Worst	1 -	5.51

Source: Compiled from Unpublished Report on Joint Study by Scottish Work Study Officers.

"The Winter Feeding of Cattle"

1959-60

Table D6

Manure Handling

	Man minutes/ton
Hand Loading	19.3
Hydraulic tractor loader	7.7
Transport 600 yards	
Horse or tractor cart	35.4
21 - 40 cwt. tractor trailer	9.8
40 cwt. tractor trailer	5.0
Trailer spreader	8.0
Setting out field heaps	13.1
Hand spreading field heaps	23.8
PTO heap spreaders	1.1
Hand spreading from trailer	14.1
Mechanical spreaders	3.0

Source: NAAS Technical Report No. 4
"Machinery & Labor in Farmyard Manure Handling".

Table D7

Straw Bale Handling

	Tractors	Men	Lifting tons/hour	Rate tons/man-hour	Extra Equipment Capital Cost per ton/man-hour
Trailer (64 bales) Mechanical loader	2	5	4.0	0.8	350
Trailer (84 bales) Mechanical loader	2	5	4.25	0.85	120
Unit load (2 x 16 bales) - Secured units	2	3	3.6	1.2	400
Unit load (2 x 16 bales) - Buckrake type	2	4	3.6	0.9	280
Trailer (45 bales) Direct thrower	2*	4*	3.25	1.1 +	230

* Baling and loading combined.

+ One man carting and two men building.

Source: A. Blythe M.A., "Straw Handling at Harvest", 1962.
Edinburgh School of Agriculture
Miscellaneous Publication No. 290.

Table D8

Silage Making

	Man minutes/ton
Buckrake	
Cut	12.6
Rake	5.2
Load and transport	27.3
Unload and ensile	27.2
Consolidate	8.5

2.8 tons/hour
2.9 men
1.9 tractors
5.7 cut/load
487 yds. travel

	Man minutes/ton
Forage Harvester*	
Cut and load	17.3
Transport	5.4
Unload	26.1
Consolidate	1.4

4.1 tons/hour
3.4 men
2 tractors
2 trailers
1392 yds. travel

* Trailers towed behind harvester.

Source: NAAS Technical Report No. 8.
"Use of Labor and Machinery in Silage Making", 1956.

Table D9

Kale

Normal spring cultivations and seeding

Singling may or may not be carried out;

May be folded for total of 40 hrs./acre,

or

Harvested for total of 100 hrs./acre.

Source: Abstracted from Moore, Ian, "Winter Keep on the Farm". London, Farmers and Stockbreeders Publications Ltd., 1960.

Table D10

Standard Time Data for Feedlot Operations

		Time in Minutes
Concentrate handling:		
1.	Pick up and fill two 5 gal. pails with chopped grain	0.132
2.	Walk per pace with two filled 5 gal. pails	0.013
3.	Walk per pace	0.012
4.	Pick up and fill one 5 gal. pail	0.077
5.	Distribute two 5 gal. pails of chop without setting down	0.080
6.	Distribute two 5 gal. pails of chop, setting one pail down	0.111
7.	Fill two 5 gal. pails with a shovel	0.619
Roughage handling:		
8.	Pick up one bale and walk (per pace)	0.019
9.	Remove twine and spread one bale by hand	0.367
10.	Distribute one bale with a fork	0.516
11.	Walk per pace with one bale and toss	0.024
12.	Walk per pace with one bale, toss and cut twine	0.027
13.	Cut two twines on bale and pull out twine	0.078
14.	Slip two twines off bale	0.093
15.	Pull out two cut twines and discard into pile	0.903
16.	Feed one bale into hammermill with a fork	1.206

Table D10

17.	Pick up a forkful of roughage	0.122
18.	Walk per pace with a forkful of roughage and dump	0.165

Unloading box operations:

19.	Mount tractor, start motor, put in gear, release clutch	0.165
20.	Mount tractor, put in gear, release clutch (motor running)	0.091
21.	Mount tractor, engage P.T.O., put in gear, release clutch (motor running)	0.167
22.	Stop tractor, turn engine off, dismount	0.117
23.	Stop tractor and dismount (engine running)	0.075
24.	Dismount tractor	0.059
25.	Hook on wagon	0.183
26.	Connect P.T.O. shaft	0.136
27.	Disconnect P.T.O. shaft	0.143
28.	Unhook wagon	0.157
29.	Climb onto feed box	0.105
30.	Climb down from feed box	0.102
31.	Spread roughage in feed box with a fork	0.410
32.	Spread grain in feed box with a shovel	0.599
33.	Position belt feed conveyor for unloading	0.083
34.	Swing belt feed conveyor for transport	0.080
35.	Position unloading auger on power box	0.103
36.	Load roughage onto feed box with 20" elevator	409 lbs/min.

Table D10

37.	Load grain onto feed box with 6" auger	465.6 lbs/min.
38.	Load one bucket of silage onto feed box using front end loader	1.716
39.	Throw one forkful of roughage on feed box	0.056
40.	Unload grain with up to 25% roughage or silage using tractor drawn unloading wagon at speed of 0.5 - 1.5 mph.	0.833-2.00 min/100 ft.
41.	Unload roughage and grain or silage and grain, with power box on truck at speed of 1.5 - 2.5 mph. (3-6 passes required per bunk)	0.417-0.750/100 ft.
42.	Mount truck, put in gear, release clutch	0.112
43.	Stop truck and dismount (engine running)	0.088

Miscellaneous operations:

44.	Unhook and open board gate	0.144
45.	Close board gate	0.112
46.	Open wire gate	0.155
47.	Close wire gate	0.265
48.	Start air-cooled motor	0.418
49.	Stop air-cooled motor	0.050
50.	Position tripod auger to bin	1.633
51.	Open bin door	0.077
52.	Close bin door	0.125
53.	Position tractor for belt work	0.560
54.	Position truck for unloading	1.117

Table D10

55.	Weight wagon and set scales for load	0.472
56.	Set scales for load	0.167

Source: Standard Time Data for Feedlot Operations
Department of Agricultural Engineering,
University of Alberta, 1962.

Table D11

Putting up Loose Hay with a Front End Mounted Stacker

The sequence of elements in the stacking operations studied was very similar in nearly all cases. Variations occurred when additional equipment such as a stack frame and a pushoff on the sweep, were used.

It should be noted that when unloading into a stack frame the sweep must be raised slightly above the height of the frame and the dumping is more or less at random until the frame is filled. Every load thereafter must be first positioned properly before it is dumped.

Field Operation	Time in Minutes
Elements	
1. Drive to windrow, lower sweep, per 10 ft. of travel	.043
2. Gather windrow, per 10 ft. of travel	.041
*3. Back away from windrow, drive forward to windrow and restart gathering	.329
4. Drive to stack with load, per 10 ft. of travel	.046

* This element is performed when hay is lost beneath the pickup teeth of the sweep while gathering the windrow. The frequency depends upon the operator's skill, the type of pickup teeth used and the condition of the hay in the windrow.

Stacking Operation

Elements

A. Building body of stack

(Operator starts to raise load as he approaches the stack)

Table D11

(i) Using a stack frame	
1. Raise load above stack frame	.193
2. Drive forward with load elevated and position load over stack	.142
3. Dump load a) with pushoff	.077
b) without pushoff	.107
4. Back away from stack, lower sweep, and turn	.209
(ii) No stack frame used	
1. Raise load per foot of lift	.021
2. Drive forward with load elevated and position load over stack	.142
3. Dump load a) with pushoff	.107
b) without pushoff	.163
4. Back away from stack and turn	.209
B. Topping the stack	
(i) Fork hay off the sweep by hand and distribute on stack, per load	3.245

Source: Standard Time Data for Feedlot Operations
Department of Agricultural Engineering,
University of Alberta, 1962.

Table D12

Filling Pit Silo with a Power Unloading Box

Elements	Time in Minutes
1. Walk to wagon from tractor	.106
2. Lower conveyor on unloading wagon	.112
3. Walk to P.T.O. drive shaft, grasp and position to tractor and slip on P.T.O. drive shaft	.468
4. Mount tractor, engage P.T.O. and drive	.167
5. Unload silage, per ton	1.089
6. Dismount	.059
7. Unhook P.T.P. shaft	.115
8. Hang up P.T.O. shaft on wagon	.108
9. Walk to tractor, per step	.012
10. Mount, put in gear, release clutch	.109

Filling Pit Silo with Dump Truck

Elements	
1. Walk to rear of truck, per step	.012
2. Remove endgate	
A. Hinged end-gate	.698
B. Chained end-gate	.164
C. Sliding end-gate	.119
(i) Carry sliding end-gate aside 10 ft.	.150
3. Walk to truck, per pace	.012

Table D12

4.	Mount truck, start engine, engage hydraulic hoist	.288
5.	Hoist load 35^o	.468
6.	Unload	.364
7.	Lower truck box	.400
8.	Replace end-gate	
	A. Hinged end-gate	.513
	B. Chained end-gate	.607
	C. Sliding end-gate	.463
9.	Walk to truck, per step	.012
10.	Mount truck, put in gear and drive	.112

Source: Standard Time Data for Feedlot Operations Department of Agricultural Engineering, University of Alberta, 1962.

APPENDIX E

PARTIAL GROSS MARGINS FOR LIVESTOCK

Prepared by J. Harkins
Edinburgh School of Agriculture

Low ground ewe

Assume 160% lambing

Lamb - 1.6 x £ 7.5	£12	
Wool	1.5	£ 13.5
Less - depreciation	- 2.0	
- concentrate fed lamb	- 1.1	
- veterinary medicine	- .75	- 3.85
		£ 9.65

Cow and 5 cwt calf off grass

Assume 90% calving

Calf - 0.9 x £45	£ 40.5	
Subsidy	8	£ 48.5
Less - depreciation	-10	
- minerals	- 1	
- veterinary medicine	- 0.75	-11.75
		£ 36.75*

Calf (5 cwt)

Return	£75	£ 75
Less - calf	-45	
- haulage and veterinary	- .75	-30.75
		£ 29.25*

Barley Beef

Return	£80	£80
Less - Calf	-19	
- Rearing to 200 lb.	- 3	
- Concentrate	-10.2	
- Losses and veterinary	- 2	-34.2
		£ 45.8*

18 Mos. Fresian

Return	£84.25	£ 84.25
Less - Calf	-15.75 (5% mort.)	
- Concentrate	-12.25	
- Milk	- 1.25	
- 2 cwt hay	- 1.2	
- Veterinary & medicine	- 1.0	-31.45
		£ 52.8*

* Add £5 as of June 17, 1963, because of higher cattle prices.

PARTIAL GROSS MARGIN FOR GRAZING

A grazing acre consists of $\frac{1}{4}$ acre each of first, second and third year of a 3-year grass ley together with $\frac{1}{4}$ acre of Italian Rye Grass.

Variable cost per acre

Seed ($\frac{1}{4}$ acre rye grass $\frac{1}{4}$ acre 3-year mixture		£1.75
S of A 5 cwt.	-65/	
K_2 0 1.8 cwt.	-42/	
Basic slag 5 cwt.	-30/	- 6.85
		£ 8.60*

* Fuel for plowing and planting is charged against preceding grain crop.

APPENDIX F

CAPACITY-COST CURVES FOR FARM TRACTORS AND MACHINES

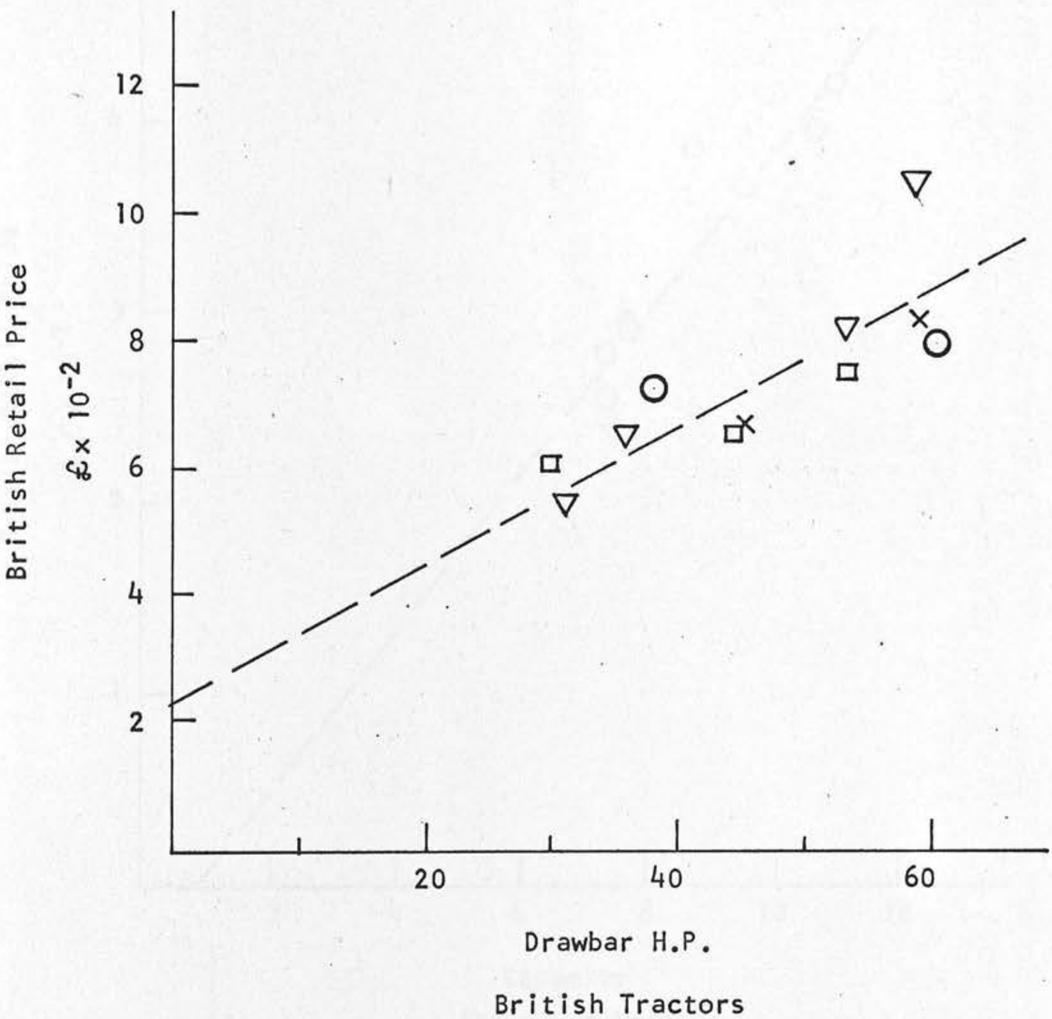
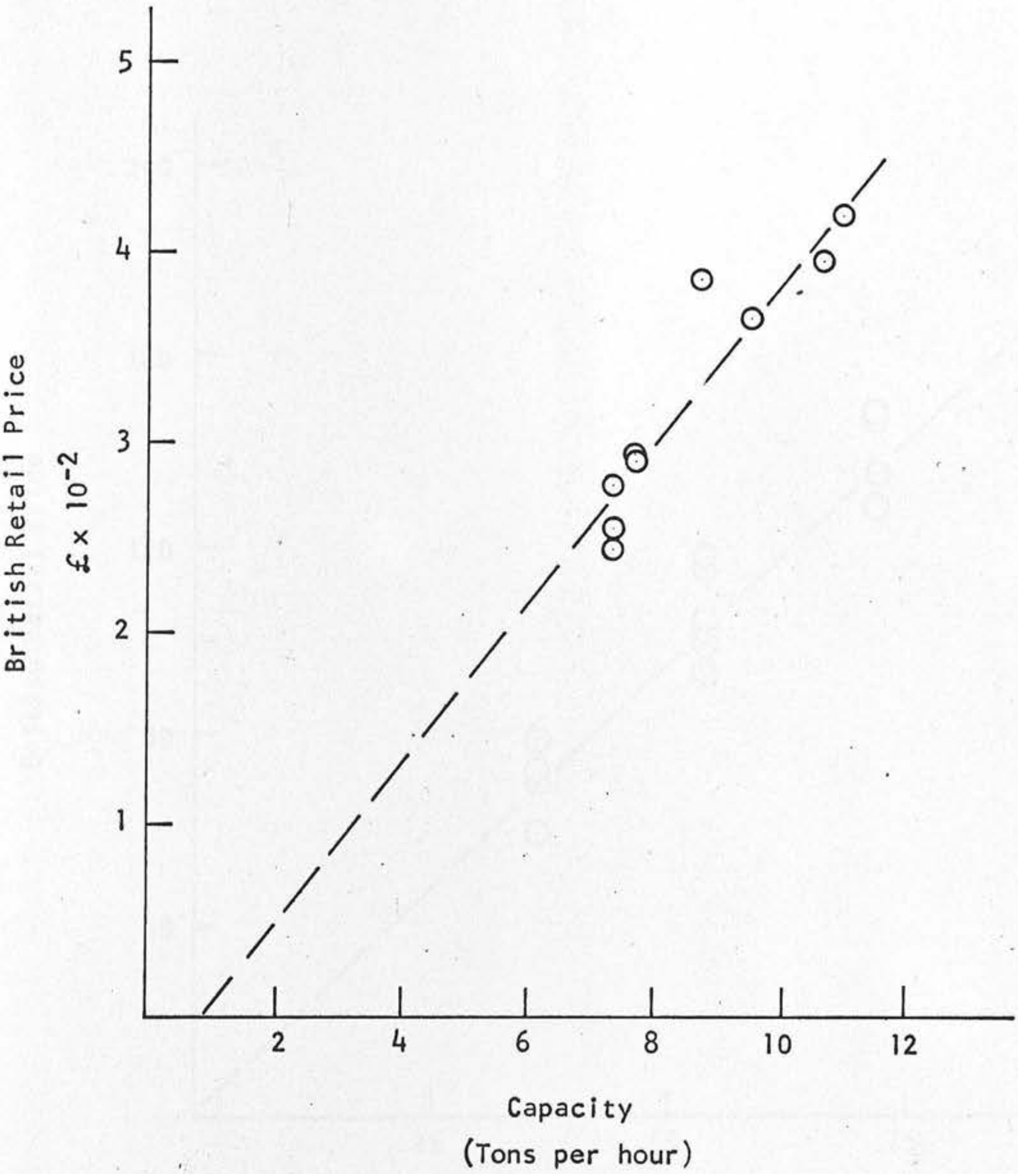
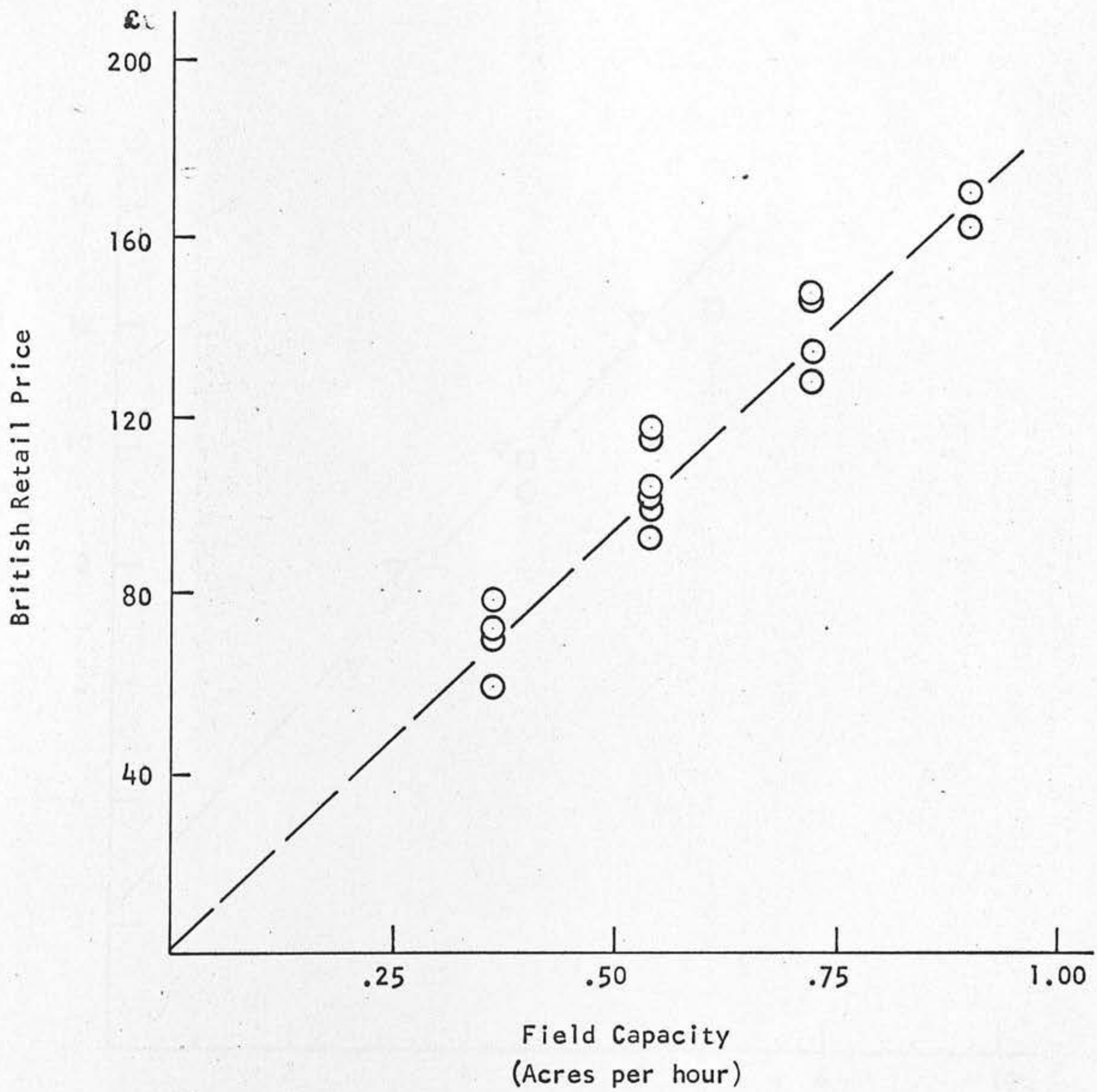


Fig. F-1



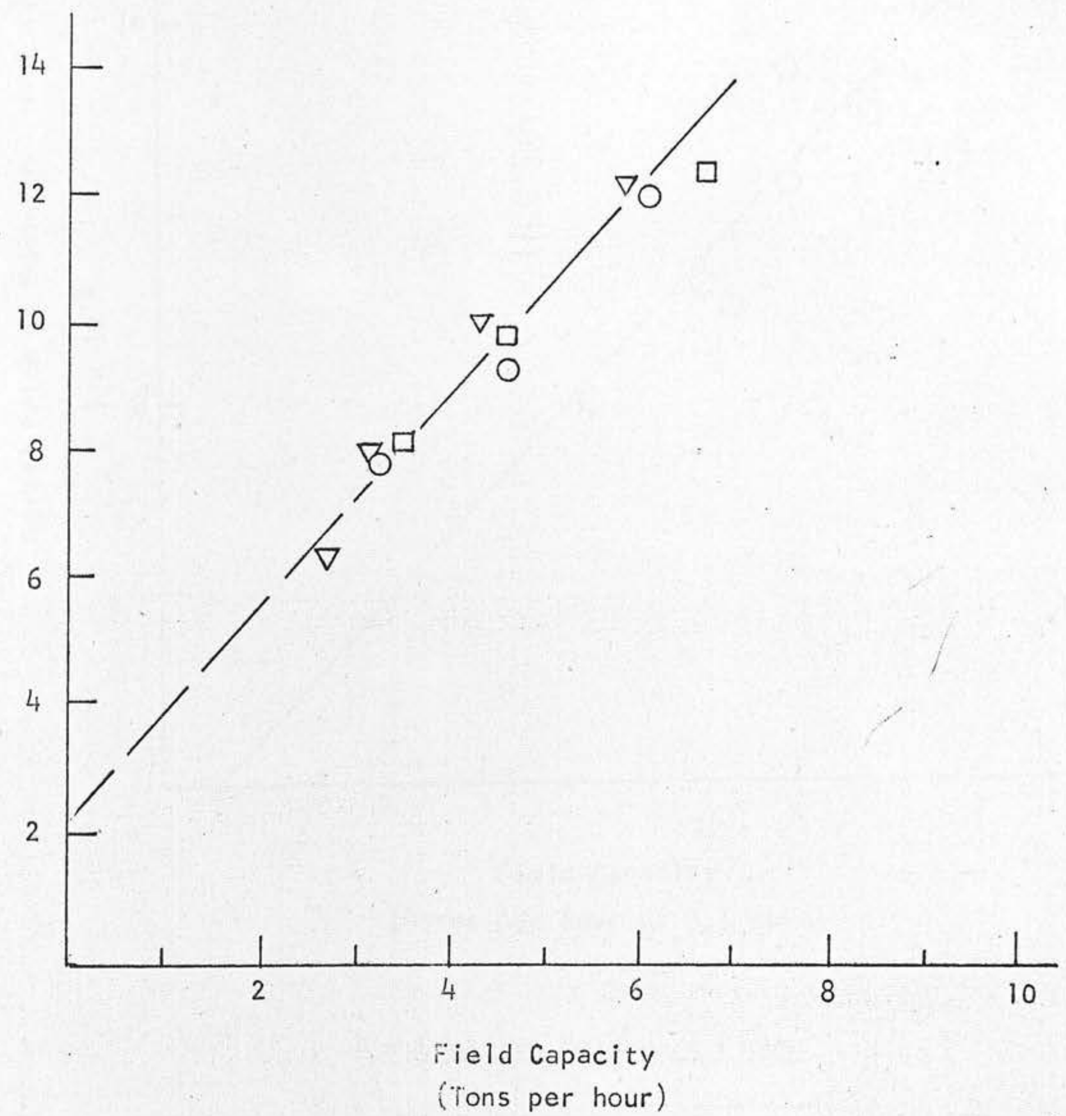
British Flail Forage Harvesters

Fig. F-2



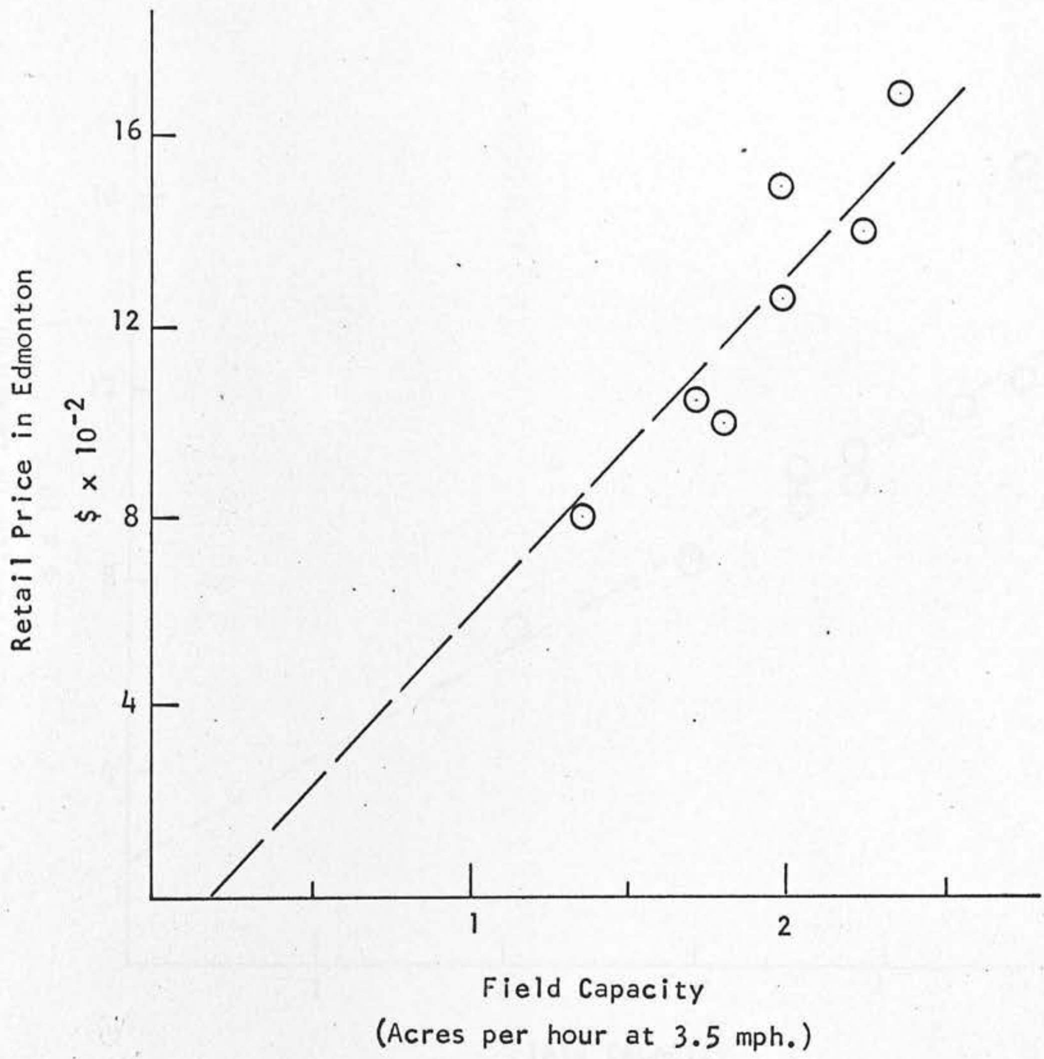
British Moldboard Plows

Fig. F-3



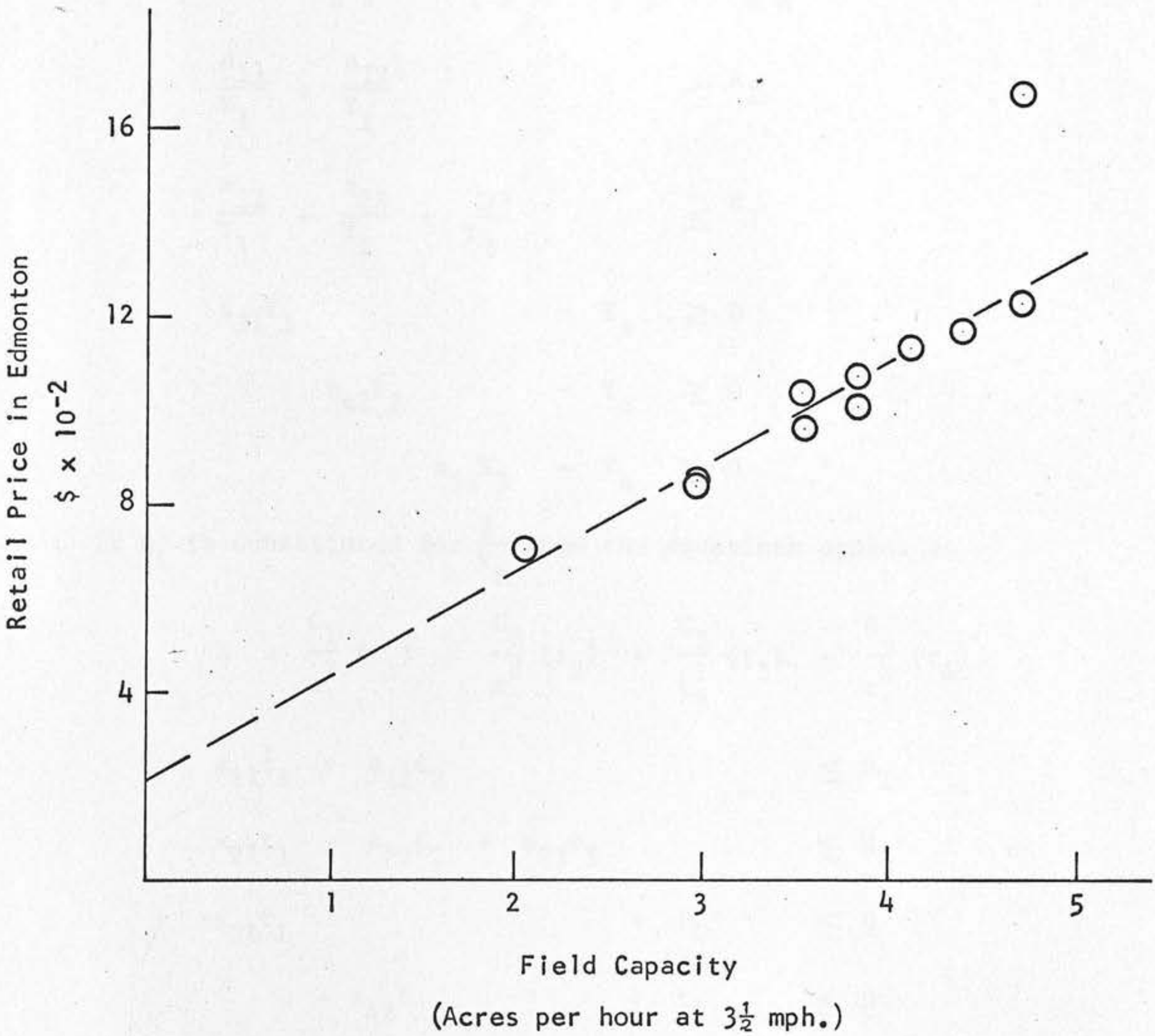
Canadian Combines

Fig. F-4



Canadian Moldboard Plows

Fig. F-5



Canadian Chisel Plows

Fig. F-6

APPENDIX G

A set of relationships were developed in Section 3.2.6 to represent equipment selection. The following example represents the function and qualifying expressions for selecting three machines and a tractor, subject to two time restrictions and the three associated power restrictions.

$$Z = C_1Y_1 + C_2Y_2 + C_3Y_3 + C_4Y_4$$

$$\frac{a_{11}}{Y_1} + \frac{a_{12}}{Y_2} \leq R_1$$

$$\frac{a_{21}}{Y_1} + \frac{a_{22}}{Y_2} + \frac{a_{23}}{Y_3} \leq R_2$$

$$a_{31}Y_1 - Y_4 \geq 0$$

$$a_{42}Y_2 - Y_4 \geq 0$$

$$a_{53}Y_3 - Y_4 \geq 0$$

If t_i is substituted for $\frac{1}{Y_i}$ then the equations appear as

$$Z = \frac{C_1}{t_1^2}(t_1) + \frac{C_2}{t_2^2}(t_2) + \frac{C_3}{t_3^2}(t_3) + \frac{C_4}{t_4^2}(t_4)$$

$$a_{11}t_1 + a_{12}t_2 \leq R_1$$

$$a_{21}t_1 + a_{22}t_2 + a_{23}t_3 \leq R_2$$

$$-a_{31}t_1 + t_4 \leq 0$$

$$-a_{42}t_2 + t_4 \leq 0$$

$$-a_{53}t_3 + t_4 \leq 0$$

The qualifying expressions are now linear but the coefficients of the function (cost-row) are non-linear, of the form

$$\left(\frac{C_i}{t_i^2}\right)$$

Jackson*, develops the general method of setting up programming problems on analog computers. Fig. G-1 illustrates the required computer diagrams, including a method for generating the non-linear cost-row coefficients. Fig. G-2 illustrates a computer developed at the Department of Agricultural Engineering, University of Alberta for linear and non-linear programming, set up to solve the problem just illustrated. Initial studies indicate that this method may offer a relatively simple approach to solving this class of non-linear problems.

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*JACKSON, ALBERT S., "Analog Computation", New York. McGraw-Hill Book Company, Inc., 1960.

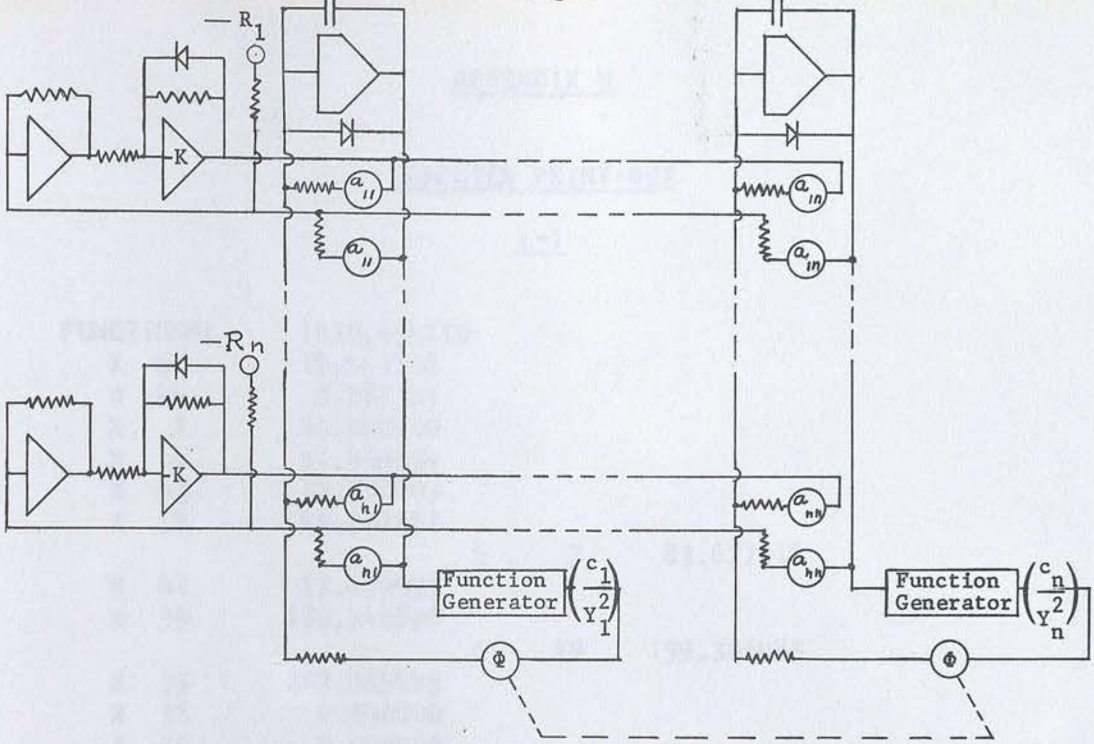

Figure G–1. Computer Diagram for Machinery Selection Problems

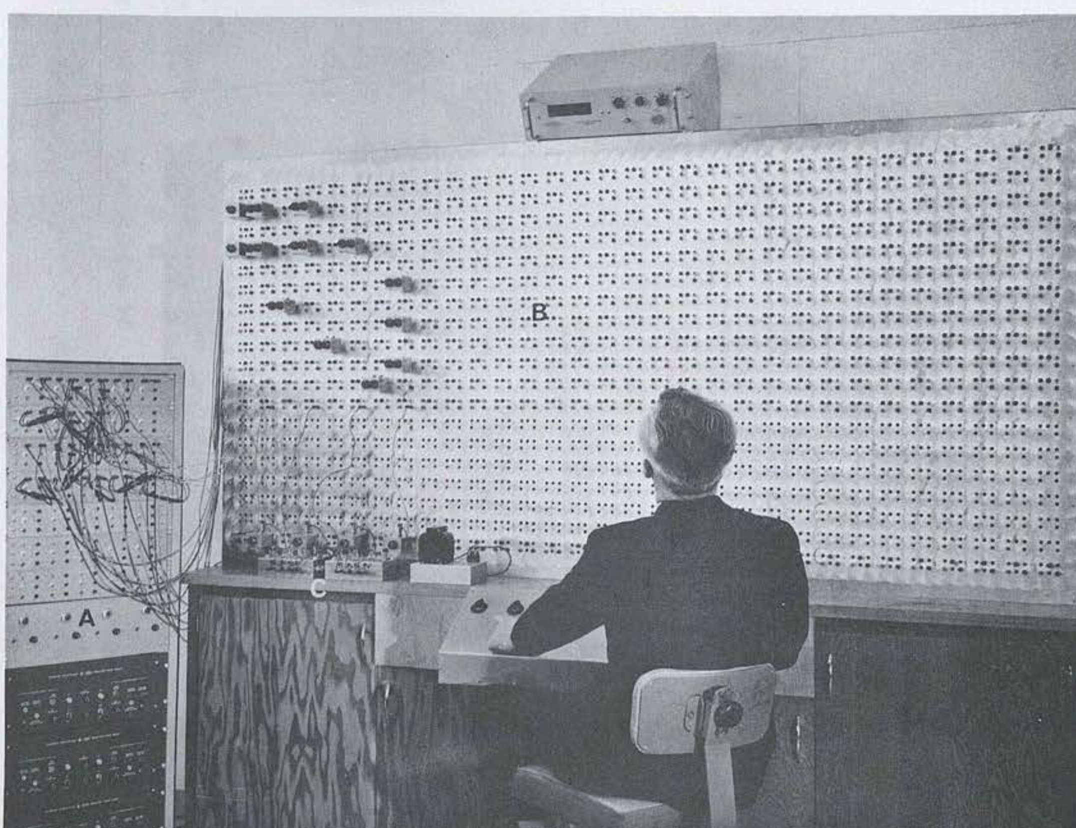

Figure G–2. Computing Equipment for Machinery Selection Problems

A – Operational amplifiers and power supplies

B – Special purpose panel

C – Diode function generators

APPENDIX H

COMPUTER PRINT-OUT

L-1

FUNCTIONAL		1430.654259			
X	56	19.463706			
X	44	0.287301			
X	1	35.000000			
X	4	51.499954			
X	47	17.700003			
X	58	45.201881			
			S	7	83.677316
X	41	17.699995			
X	39	180.742898			
			S	10	159.306485
X	35	287.885445			
X	38	0.000000			
X	30	0.000000			
			S	14	0.000000
			S	15	0.000000
X	26	0.000000			
X	37	0.000000			
			S	18	0.000000
X	27	0.000000			
X	8	14.577218			
			S	21	0.000000
X	28	0.000000			
X	63	25.929531			
X	5	51.500046			
X	11	6.161274			
X	50	2.432961			
X	46	23.499993			
			S	12	14.901851
X	40	9.222775			
X	49	18.921181			
			S	31	177.000000
			S	32	42.633820
			S	33	177.000000
X	64	8.623966			
			S	35	174.126994
			S	36	250.794705
			S	37	65.393698
			S	38	235.000000
X	12	89.838726			
X	9	9.422824			
X	13	24.000000			
X	57	111.359645			
X	6	16.000000			
			S	44	120.000000
			S	45	266.968585
			S	46	0.000000

X	0	1430.654259
S	3	0.247852
X	2	0.066620
X	3	1.812136
S	4	1.251566
S	24	0.226809
S	43	0.015000
X	7	2.043023
X	10	2.364567
S	40	0.020998
S	20	0.009464
S	6	0.881585
S	39	0.050000
S	41	0.049998
S	42	0.021001
X	15	0.199254
X	16	0.110746
X	17	1.872012
X	18	0.688762
X	19	0.914372
X	20	0.005666
X	21	0.394360
X	22	0.320245
X	23	0.233505
X	24	0.106527
X	25	0.130740
S	16	0.000291
S	19	0.020289
S	22	0.016477
X	29	0.116721
S	13	0.118423
X	31	0.022250
X	32	0.398471
X	33	0.293320
X	34	0.313371
S	11	0.235345
X	36	3.587368
S	17	0.011653
X	55	0.350000
S	9	1.889254
S	29	0.020999
S	8	1.469999
X	42	0.140000
X	43	0.290000
S	2	0.391003
X	45	0.290000
S	27	0.050006
S	5	1.989498

X	48	0.140000
S	30	0.014999
S	26	0.084999
X	51	0.200000
X	52	0.640000
X	53	0.060000
X	54	0.850000
S	28	0.049998
S	1	1.580083
X	14	2.307429
S	25	0.268734
X	59	0.350000
X	60	0.500000
X	61	0.210000
X	62	0.500000
S	23	2.699757
X	34	0.029000
X	65	0.350000

COMPUTER PRINT-OUT

L-2

FUNCTIONAL		3298.549402			
X	41	169.689017			
X	49	120.821041			
X	42	30.500000			
X	53	76.218884			
X	12	213.994947			
X	11	20.924031			
			S	13	0.000000
X	3	65.625073			
			S	11	0.000000
X	19	0.000000			
X	21	0.000000			
X	40	1427.646299			
X	25	0.000000			
			S	14	0.000000
X	15	0.000000			
X	28	124.223710			
			S	17	178.990939
X	10	44.700955			
X	35	8.486336			
X	43	23.233739			
			S	21	0.000000
			S	22	0.000000
			S	23	229.674960
			S	24	178.803379
			S	25	296.570884
X	16	114.837423			
X	45	10.321211			
			S	28	0.000000
X	30	1.357421			
X	36	20.903162			
X	33	101.351998			
			S	32	157.626486
X	55	209.055505			
X	38	225.575540			
			S	30	74.271632
X	54	30.500000			
			S	20	125.801199
			S	38	15.062424
			S	39	30.500000
X	4	65.625083			
X	48	30.500000			
X	17	89.401568			
			S	43	39.000019
			S	44	39.000000

X	7	328.125381
X	52	39.000000
X	1	65.625073
X	44	30.500000
S	49	27.031281
S	50	25.000000
X	31	18.374044
S	52	0.000000

X	0	3298.549402
S	47	0.500000
X	2	6.123562
S	8	1.117190
X	6	3.152480
X	5	1.576241
S	40	0.500000
S	45	0.500000
S	37	0.125000
X	9	1.706183
S	18	0.111990
S	34	0.500000
S	5	4.706183
S	9	0.111979
X	37	0.450001
S	15	0.111984
S	26	0.013066
S	7	3.868766
S	10	0.024321
X	8	1.151970
X	20	3.619685
X	13	0.624009
X	22	0.354177
X	23	0.159937
X	24	3.726114
X	14	0.383257
X	26	0.099821
X	27	0.232950
S	29	0.000000
X	29	3.619682
S	16	0.024321
S	51	0.158173
S	35	0.125000
S	31	0.070000
X	34	3.619687
S	19	0.024308
X	18	0.232945
S	42	2.140001

X	32	0.232950
X	39	0.082810
S	12	0.126120
S	1	57.214344
S	3	4.497919
S	6	3.813759
S	48	0.125000
S	27	5.360086
X	46	0.375000
X	47	0.500000
S	41	0.125000
S	2	11.213556
X	50	0.375000
X	51	0.500000
S	46	0.125000
S	4	5.089999
S	36	0.500000
S	33	0.500000
X	56	0.375000
X	57	0.500000

COMPUTER PRINT-OUT

L-3

(No Cow Minimum)

FUNCTIONAL	2582.259626			
X 37	2.223061			
X 44	30.731360			
X 7	49.999874			
X 43	23.675785			
X 22	44.132937			
X 17	53.063838			
X 28	5.498157			
		S	15	378.586047
X 33	7.002281			
X 23	25.225219			
X 24	6.885076			
X 38	9.236481			
		S	13	16.362685
X 2	33.316848			
		S	7	64.141463
X 30	43.745737			
X 27	2.522447			
X 3	50.000023			
		S	17	84.274090
X 25	6.885076			
X 18	291.662170			
X 14	138.503466			
X 39	4.759301			
		S	24	166.666620
X 11	173.333938			
X 10	60.000000			
		S	16	203.897347
X 20	120.495777			
X 9	33.318143			
X 32	4.413286			
		S	10	53.618880
		S	25	125.001022
X 19	153.657054			
		S	34	18.593727
		S	14	630.018169
X 4	50.000023			
		S	37	0.000000

X	0	2582.259626
A	27	-0.140004
S	9	0.016900
X	6	2.878788
A	36	0.000000
X	5	2.440381
S	4	7.000387
S	35	3.136972
S	28	0.000000
S	21	2.305057
S	33	2.860607
X	28	0.164780
X	12	0.159809
S	6	0.018709
X	27	0.160114
S	11	0.101384
S	18	0.081256
X	18	0.114199
S	3	4.121571
X	23	0.011942
S	30	0.295155
S	31	5.080927
S	8	0.109732
X	1	5.183336
S	5	0.108664
S	12	0.026000
X	26	0.269718
S	32	0.463698
S	19	0.156235
X	29	0.201245
X	35	0.140004
S	23	2.000000
S	22	2.361506
S	26	4.020004
S	29	0.301175
S	1	4.404814
X	36	0.150000
X	41	0.086168
X	30	0.202244
S	2	5.901283
S	20	1.681212
X	22	0.161079
X	42	0.027637
X	43	0.035131
X	31	0.559705

COMPUTER PRINT-OUT

L-3

(50 Cow Minimum)

FUNCTIONAL		2502.841276			
X	17	55.916171			
X	44	40.236964			
			S	15	461.966015
X	14	255.793183			
X	37	4.294127			
X	20	94.266815			
			S	6	95.791165
X	33	12.500331			
			S	10	77.100887
X	23	37.525510			
X	24	50.000000			
X	38	83.419580			
			S	13	158.091198
X	2	33.334640			
			S	7	6.587737
			S	14	784.894595
X	27	3.752449			
X	3	50.000012			
			S	17	98.464533
X	4	18.919950			
X	18	252.798436			
X	9	64.410490			
X	36	50.340347			
			S	24	166.660386
X	11	142.244369			
X	10	60.000000			
X	43	11.449049			
X	22	3.411664			
X	7	50.000012			
X	32	0.341088			
X	39	83.464359			
			S	16	216.882390
X	19	201.187006			
			S	34	21.313988
			S	25	86.138936
X	5	31.080322			
			S	37	0.000000

X	0	2502.841276
A	27	-0.139988
X	22	0.247166
X	6	2.878479
S	20	4.121529
A	36	-3.050667
S	4	6.999987
S	29	0.130416
S	28	0.000000
S	21	4.745391
S	33	2.860540
X	1	0.303103
X	12	0.159853
X	17	1.669368
X	31	3.000244
S	11	0.101387
X	35	0.139988
X	18	1.941494
S	3	4.121529
X	19	0.981179
S	30	0.301707
S	9	0.028210
S	8	0.087619
X	27	0.159942
S	5	0.064636
S	12	0.026053
X	26	0.269894
S	32	0.463687
S	19	0.156206
X	29	0.201225
X	28	0.165000
S	23	2.000000
S	22	2.361266
S	26	4.019988
S	31	4.422738
S	1	4.795170
X	36	0.150000
X	30	0.202226
S	18	0.081225
S	2	5.876606
X	40	3.050667
X	41	0.696258
X	42	0.142595
X	23	0.180629
S	35	3.136861

COMPUTER PRINT-OUT

L-4

FUNCTIONAL 2409.411749

			S	36	109.200029
			S	5	35.000000
X	30	0.000000			
X	5	0.000000			
			S	34	155.056534
X	12	74.983870			
X	13	488.432498			
X	22	48.843259			
X	11	13.353598			
X	44	2303.887006			
X	43	668.133234			
X	36	174.282207			
			S	13	2849.478769
			S	14	568.160791
X	18	94.983870			
X	20	56.677863			
			S	17	86.569493
X	40	513.074161			
			S	19	0.000000
			S	20	0.000000
X	26	0.000000			
X	39	345.555254			
			S	24	0.000000
X	38	0.000000			
			S	25	0.000000
			S	26	0.000000
			S	38	579.353430
			S	28	34.529977
X	3	88.336767			
X	7	278.868804			
X	41	353.026441			
X	42	190.301975			
X	8	74.477804			
			S	3	204.391002
			S	35	315.099451
X	31	0.000000			
X	6	88.337428			
			S	38	0.000000

X	0	2409.411749
X	1	0.020000
X	2	2.159904
S	29	0.370000
X	37	0.150930
X	4	1.590437
S	4	2.310402
S	30	0.370000
S	37	0.000000
X	9	0.144796
X	23	0.061368
S	6	2.310402
S	7	3.923865
X	14	0.024722
X	15	0.842143
X	16	0.350000
X	17	1.530349
S	15	0.105020
X	19	0.144794
S	16	0.011848
X	21	1.694031
X	28	0.183619
S	9	1.264525
X	24	0.842144
S	8	0.306065
S	21	0.075046
X	27	0.144794
X	29	1.917875
S	22	0.031404
X	25	1.530354
X	48	0.819935
X	32	0.717355
X	33	2.385328
X	34	1.309997
X	35	0.139099
S	12	0.063229
S	2	3.385226
S	24	0.071192
X	47	0.740000
S	18	0.063229
S	37	0.370000
S	32	0.550065
S	17	1.834796
S	10	3.502760
S	27	4.545304
X	46	0.740000
X	45	0.740000
S	1	30.575680
X	49	0.165204

COMPUTER PRINT-OUT

L-5

(Without Silo)

FUNCTIONAL		2087.654546			
X	33	31,568362			
X	30	25.000000			
X	40	10.650757			
X	6	86.142692			
X	2	57.142692			
			S	6	60.299876
X	14	11.485688			
X	19	120.000000			
X	17	0.000000			
			S	10	0.000000
			S	11	0.000000
X	18	114.856877			
X	13	57.215815			
			S	14	93.428236
X	29	29.499950			
X	28	3.046910			
X	4	14.489482			
			S	3	256.734500
			S	19	65.357223
X	20	120.000000			
			S	21	984.282046
X	22	72.000043			
X	23	57.142692			
X	32	150.236227			
			S	25	231.428974
			S	26	250.000000
			S	27	399.785865
			S	28	250.000000
			S	29	185.457572
			S	30	250.000000
			S	31	590.000000
X	21	94.642777			
			S	33	264.530903
X	5	271.224236			
X	24	24.999964			
X	1	0.000000			
X	39	39.446912			
			S	38	284.857519
			S	39	150.000000
			S	40	579.857519
X	15	92.217656			
X	10	0.000000			
			S	43	0.000000

X	0	2087.654546
S	36	0.049998
X	3	7.911141
A	5	0.000000
S	17	0.031000
S	34	0.050000
S	4	0.114434
X	7	0.324425
X	8	0.179903
X	9	0.130097
S	42	0.121824
X	11	0.061200
X	12	0.104837
S	13	0.027553
S	7	0.144425
S	41	0.121193
X	16	2.254393
S	9	0.119097
S	12	0.091550
S	8	1.869903
S	20	4.354393
S	32	0.029000
S	22	0.500000
S	23	0.049996
S	35	0.021000
X	25	0.290000
X	26	0.290000
X	27	0.290000
S	16	0.823821
S	15	0.898821
S	2	3.469987
X	31	0.290000
S	24	0.020996
S	1	28.513283
X	34	0.500000
X	35	0.500000
X	36	0.500000
X	37	0.500000
X	38	0.210000
S	37	0.020999
S	18	0.017000
X	41	0.500000

COMPUTER PRINT-OUT

L-5

(With Silo)

FUNCTIONAL		2209.283345			
X	40	11.425929			
X	39	39.625737			
			S	3	252.759847
X	6	60.524167			
X	28	2.808513			
X	5	269.236866			
X	7	25.618672			
X	10	82.591910			
X	17	108.078939			
			S	10	776.399661
X	11	43.944914			
			S	12	0.011693
X	18	0.000000			
			S	14	0.000000
X	23	31.524167			
			S	30	245.880218
X	2	57.142809			
X	4	16.477016			
			S	19	85.598020
X	29	29.500017			
			S	21	984.286215
X	30	24.999993			
X	32	149.759570			
X	19	120.000000			
			S	25	231.428588
			S	26	250.000000
			S	27	412.595168
			S	28	250.000000
X	22	72.000083			
X	24	25.000000			
			S	31	590.000000
X	21	74.401980			
			S	33	266.914869
X	27	0.412019			
X	33	31.290260			
X	1	0.000000			
X	20	120.000000			
			S	38	283.466057
			S	39	150.000000
			S	40	578.466057
			S	41	0.000578
			S	42	331.370870
			S	43	0.000000

X	0	2209.283345
S	36	0.050013
S	17	0.031000
A	5	0.000000
S	1	30.225938
S	6	0.121843
S	4	1.446541
S	7	1.408683
X	8	0.179878
X	9	0.130122
S	8	1.869878
S	11	0.430028
X	12	0.108939
X	14	1.364982
X	13	1.441158
X	15	0.066560
X	16	3.586515
S	9	0.183719
S	18	0.017000
S	24	0.021001
X	36	0.209987
S	32	0.029001
S	29	0.028999
S	15	0.898828
S	16	0.823827
X	25	0.290000
X	26	0.290000
S	34	0.049991
X	3	5.246929
S	20	5.686515
S	22	0.500000
X	31	0.290000
S	23	0.050001
S	35	0.021002
X	34	0.500000
X	35	0.500000
S	37	0.021000
X	37	0.500000
X	38	0.210000
S	2	3.470011
S	13	0.027310
X	41	0.500000

COMPUTER PRINT-OUT

L-6

(1-Man Working Force)

FUNCTIONAL		1238.0597			
X	6	74.6666			
X	26	8.7514			
X	35	5.0659			
X	18	9.5663			
X	11	30.0000			
X	14	24.5551			
X	15	11.4072			
X	16	42.0081			
X	17	37.8527			
			S	10	350.7668
X	32	54.5801			
X	21	9.9940			
X	12	59.5506			
			S	13	128.6678
X	13	28.4493			
			S	16	207.8133
X	33	21.4634			
X	9	195.0000			
X	7	66.7043			
X	10	50.0000			
X	5	1.6666			
			S	22	.0000
			S	23	.0000
			S	24	.0000
X	4	108.3333			
			S	26	.0000
X	3	108.3333			
X	1	43.3333			
			S	29	.0000
X	34	1.3211			
X	28	.8269			
X	29	4.1035			
X	30	4.1035			
X	25	7.8412			
			S	35	.0000
X	23	20.8893			
X	31	6.1553			

X	0	1238.0597
S	28	.1097
S	36	.2238
S	27	.1332
S	25	.2750
S	21	.3668
S	1	4.0942
A	19	999.9990
X	8	.3100
S	18	2.0000
S	20	.5000
S	5	1.3514
S	14	.5273
S	15	.2009
S	6	.0270
S	7	.0160
S	4	.0165
X	19	.2174
A	4	999.9989
S	9	.0508
X	20	.1375
S	12	.2009
X	22	1.4940
X	2	6.1254
X	24	.0919
S	34	.1458
X	2	2.3396
X	27	.0919
S	31	.0304
S	32	.0303
S	33	.0205
S	37	.00527
A	11	999.9991
S	17	.5273
S	13	.1536
S	3	4.6071

COMPUTER PRINT-OUT

L-6

(3-Man Working Force)

FUNCTIONAL		1440.9345			
X	28	3.4059			
X	4	18.6537			
X	34	8.5364			
X	6	159.2308			
X	25	6.0433			
X	2	21.1539			
X	19	29.5207			
X	20	12.0528			
X	21	57.9063			
X	16	30.0000			
X	30	6.8516			
X	22	45.5945			
			S	13	428.8235
X	50	.1356			
X	24	6.4999			
X	29	7.0118			
X	17	53.9373			
			S	11	7.4447
			S	17	231.0118
X	18	34.0626			
			S	21	303.2563
X	44	20.0000			
			S	41	37.2748
X	39	.5457			
X	9	265.7581			
X	15	50.0000			
X	13	207.0000			
			S	28	292.0611
			S	29	457.9956
			S	30	331.8687
X	3	18.6537			
X	36	7.8376			
			S	33	199.2845
			S	34	330.6576
X	43	36.02			
			S	36	159.2307
			S	37	159.2307
X	8	194.9999			
X	45	1.6083			
X	49	.2324			

X	35	4.4281			
X	46	2.8951			
X	40	2.4224			
X	42	3.6336			
X	48	.1544			
X	41	2.4224			
X	32	11.7753			
X	47	.4374			
X	12	18.5980	S	50	18.5980
X	10	18.5980			

X	0	1440.9345
S	18	.5273
S	38	.0908
S	31	.3190
S	2	5.1552
S	47	.2122
S	4	5.1555
X	7	.0989
S	6	1.2015
A	25	999.9989
S	51	.2000
X	11	.1769
S	49	7.3000
S	27	.7158
X	14	.2158
S	26	.7158
S	23	.1978
S	19	.1978
S	20	.2009
S	7	.0270
S	8	.0160
S	9	.0165
X	26	.4336
X	23	1.0384
S	15	.1350
A	5	999.9990
S	12	.0508
X	27	.1375
S	1	-4.9231
S	16	.2009
X	5	.0654
X	31	1,4939
X	1	1.9866
X	33	.0919
S	10	.2065

X	37	.0000
S	32	.1359
S	3	2.3398
X	38	.0919
S	24	2.0000
S	43	.0096
S	46	.0779
S	44	.0139
S	35	.1400
S	22	.5273
S	39	.0493
S	42	.0139
S	48	.2750
S	45	.1300
S	40	.5669
A	14	999.9990